

Math 21A
 Kouba
 Discussion Sheet 5

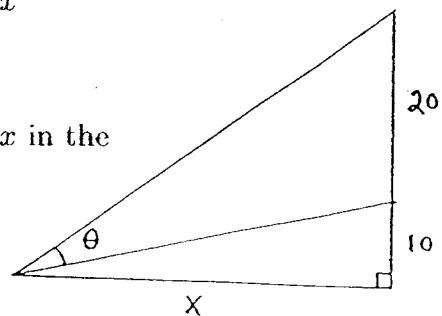
1.) Use any method to differentiate the following functions. You need not simplify answers.

a.) $y = 7 \sec 3x$ b.) $f(x) = \frac{x \sin x}{1 + \tan x}$ c.) $f(x) = \left(\frac{3x + 7}{7x - 9}\right)^{50}$

d.) $g(x) = x^3 \cos x^2$ e.) $f(x) = \sqrt{1 + \sqrt{2 + \sqrt{3 - x}}}$

f.) $y = \cot^5(\sin^3(10x^5))$

2.) Derive a formula for the measure of angle θ as a function of x in the given diagram.



3.) Let $f(x) = x(x - 5)^4$.

- a.) Solve $f'(x) = 0$ for x .
- b.) Solve $f''(x) = 0$ for x .

4.) Consider the function $f(x) = 1 - x^{2/3}$ on the interval $[-1, 1]$. Show that $f(1) = f(-1) = 0$, but that $f'(x)$ is never zero on the interval $[-1, 1]$. Explain how this is possible, in view of Rolle's Theorem.

5.) Let $f(x) = \begin{cases} -x^2, & \text{if } -1 \leq x \leq 0 \\ x^2(x - 1), & \text{if } 0 < x \leq 2. \end{cases}$

- a.) Sketch the graph of f .
- b.) Show that f satisfies the conditions of the Mean Value Theorem (MVT) over the interval $[0, 2]$, including special attention at $x = 0$, and determine all values of c guaranteed by the MVT.

6.) Find all points (x, y) which are equidistant from the three given points $(0, 0)$, $(4, 0)$, and $(3, 2)$.

7.) Derive an equation of the line tangent to the graph of $y = \frac{x}{x^2 + 3}$ at $x = -1$.

8.) Derive equations of all lines which are tangent to the graph of $y = -7 - x^2$ and passing through the point $(3, 0)$.

9.) Assume that a baseball is projected directly upward from the ground with an initial velocity of 112 ft./sec. Assuming only that the acceleration due to gravity is -32 ft./sec.^2 , derive equations for the height of the ball above the ground after t seconds and the velocity of the ball after t seconds. What is the velocity of the ball after $t = 1 \text{ sec.}$, $t = 2 \text{ sec.}$, and $t = 3 \text{ sec.}$? At what time does the ball reach its maximum height? What is the

maximum height ? In how many seconds does the ball strike the ground ? What is the ball's velocity as it strikes the ground ?

10.) A spaceship is traveling (left to right) along the curve $y = 3 \cos x$. An object is released from the spaceship at $x = \frac{\pi}{3}$ and travels along a line tangent to the graph of $y = 3 \cos x$

towards the x-axis.

a.) At what point x will the object strike the x-axis ?

b.) At what angle θ will the object strike the x-axis ?

+++++

The following problem is for recreational purposes only.

11.) Assume that you have three boxes labeled and filled with fruit. One box contains APPLES only. One box contains ORANGES only. One box contains APPLES and ORANGES. Unfortunately, ALL of the boxes are labeled incorrectly. Explain how to correctly relabel all of the boxes by (without peeking into any box) selecting exactly one fruit from exactly one box.