

Math 21A  
Kouba  
Discussion Sheet 7

1.) Use the Intermediate Value Theorem to verify that the following equation is solvable, then use Newton's Method to estimate the value of the solution to four decimal places :  
 $(x - 1)^3 = 10 + \sqrt{x}$

2.) Find the slope and concavity of the graph of  $xy^2 + y = 2$  at  $x = 0$  and at  $x = 1$ . Sketch the graph of this equation.

3.) The volume  $V$  of a sphere is changing at the rate of  $\pi$  ft.<sup>3</sup>/min. At what rate is the sphere's surface area  $S$  changing when

a.)  $S = 4\pi$  ft.<sup>2</sup> ?      b.)  $S = 36\pi$  ft.<sup>2</sup> ?

4.) Find the height  $h$  and radius  $r$  of a right circular cone of maximum volume which can be inscribed in a sphere of radius 1 ft.

5.) Car B is 30 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph.

a.) At what rate is the distance between the cars changing after  $t = \frac{1}{5}$  hr. ?  $t = \frac{1}{3}$  hr. ?

b.) What is the minimum distance between the cars and at what time  $t$  does the minimum distance occur ?

6.) Assume that  $x^2 + (5 - y)^3 = 2x + 125$ .

a.) Find  $y' = \frac{dy}{dx}$  at the point  $(0, 0)$  using implicit differentiation.

b.) Solve the original equation for  $y$ . Then find  $y' = \frac{dy}{dx}$  at the point  $(0, 0)$  using ordinary differentiation.

7.) A conical tank (point down) has height 10 ft. and base radius 8 ft. Water begins flowing into the tank at the rate of  $\pi$  ft.<sup>3</sup>/sec. At what rate is the depth  $h$  of the water changing

a.) when  $h = 1$  ft. ?      b.) when  $h = 9$  ft. ?

8.) Assume that  $xy^2 = x + \tan y$ . Write  $y'' = \frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$  only.

9.) Use differentials to estimate the value of  $(9900)^{\frac{1}{4}}$ .

10.) The radius of a sphere is measured with absolute percentage error of at most 4%. Use differentials to estimate the maximum absolute percentage error in computing the sphere's

a.) surface area.

b.) volume.

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The following problem is for recreational purposes only.

11.) Write a formula which will determine the  $n$ th term in the following list for  $n = 1, 2, 3, 4, 5, 6, \dots$ . What is the 137th number in this list ?

5, 9, 13, 17, 21, 25, . . .