

Math 21A  
 Kouba  
 Discussion Sheet 9

1.) Consider function  $f(x) = (\ln(\frac{1}{4}x^4 + 1))^5$  for  $x \leq -1$ . Show that  $f$  is one-to-one and find its inverse function.

2.) Find all values of  $K$  for which the function  $f(x) = -2x^3 + 3Kx^2$  is one-to-one.

3.) a.) Show that  $\log_B x = \frac{\ln x}{\ln B}$ .

b.) Assume that  $0 < B < 1$  and compute

i.)  $\lim_{x \rightarrow \infty} \log_B x$     ii.)  $\lim_{x \rightarrow 0^+} \log_B x$

4.) Compute the following limits.

a.)  $\lim_{n \rightarrow \infty} \left(1 - \frac{7}{n^2}\right)^n$     b.)  $\lim_{n \rightarrow -\infty} \left(1 + \frac{13}{n}\right)^{n^2}$     c.)  $\lim_{x \rightarrow 0} \left(\frac{x + 2x^2}{x}\right)^{5/x}$

d.)  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$     e.)  $\lim_{h \rightarrow 0} \frac{\sin 3h - 3h}{2h^2}$     f.)  $\lim_{x \rightarrow 0} \frac{(e^x - 1)^2}{\sin x^2}$

g.)  $\lim_{x \rightarrow 0} \frac{e^{-1/x^2}}{x}$     h.)  $\lim_{x \rightarrow \infty} (\ln x)^{1/x}$     i.)  $\lim_{x \rightarrow 0} (x)^{\tan x}$

j.)  $\lim_{n \rightarrow \infty} (1 + n)^{1/n}$     k.)  $\lim_{n \rightarrow \infty} (3^n + 4^n)^{1/n}$

5.) Find  $y' = \frac{dy}{dx}$ . Do not simplify your answers.

a.)  $y^3 = x^x y^2$     b.)  $y = x^{(y^{\tan y})}$     c.)  $y = \ln \left( \frac{x \cdot \sin x \cdot \cos(2x)}{(x + 2)^x \cdot \sqrt{\tan(3x)}} \right)$

d.)  $y = \frac{x^2(x + 1)^3(x - 1)^5}{(x - 2)(x + 3)^4(2x - 5)^7}$     e.)  $x^2 e^{xy} + \ln y = x^3 - 4y^3$

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The following problem is for recreational purposes only.

6.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. What is the total distance the bumblebee travels by the time the riders meet ?