Section 4.3

190:5 \[ Y = 2x^3 + x + 2 \rightarrow Y' = 6x^2 + 1 \rightarrow Y'' = 12x \]

190:7 \[ Y = \frac{x}{x+1} \rightarrow Y' = (x+1)' \cdot x' - x' \cdot (x+1)'' \]
\[ (x+1)^2 \rightarrow Y'' = -2 \left( \frac{2}{(x+1)^3} \right) \]

190:9 \[ Y = x \cos x \rightarrow Y' = x \cdot -\sin x + \cos x = \cos x - x \sin x \]
\[ \rightarrow Y'' = -\sin x - (x \cos x + \sin x) = -2 \sin x - x \cos x \]

190:10 \[ Y = \sec x \rightarrow Y' = \sec x \tan x \rightarrow \]
\[ Y'' = \sec x \cdot \sec^2 x + \sec x \cdot \tan x \cdot \tan x \]
\[ = \sec^3 x + \sec x \cdot \tan^2 x = \sec^3 x + \sec x \cdot (\sec^2 x - 1) \]
\[ = 2 \sec^3 x - \sec x \]

190:12 \[ Y = \frac{x}{\tan x} \rightarrow Y' = \frac{\tan x - x \cdot \sec^2 x}{\tan^2 x} \]
\[ = \frac{\tan x}{\tan^2 x} - \frac{x \cdot \sec^2 x}{\tan^2 x} = \cot x - x \cdot \csc^2 x \rightarrow \]
\[ Y'' = -\csc^2 x - \left[ x \cdot 2 \csc x \cdot -\csc x \cot x + \csc^2 x \right] \]
\[ = -\csc^2 x + 2x \csc^2 x \cot x - \csc^2 x \]
\[ = -2 \csc^2 x + 2x \csc^2 x \cot x \]

190:15 \[ Y = \sin 3x \rightarrow Y' = 3 \cos 3x \rightarrow Y'' = -9 \sin 3x \]

190:16 \[ Y = \tan x^2 \rightarrow Y' = \sec^2 x^2 \cdot 2x = 2x \cdot \sec^2 x^2 \rightarrow \]
\[ Y'' = 2x \cdot 2 \sec^2 x^2 \cdot \sec^2 x^2 \cdot \tan x^2 \cdot 2x + 2 \sec^2 x^2 \]
\[ = 8x^2 \sec^2 x^2 \cdot \tan x^2 + 2 \sec^2 x^2 \]

190:18

The price goes higher but the slope (derivative) is smaller.
Let $s(t)$ be distance (miles) from airport at time $t$ (hours);

let $a$ be acceleration of plane;

let $T$ be total time to fly 120 miles,

then $s(0) = 120$ mi., $s'(0) = -500$ mph \( \Rightarrow \) $s(t)$ is decreasing!

Thus, acceleration

$$s''(t) = a \quad \text{velocity} \quad s'(t) = At + c$$

(and $s'(0) = -500 \Rightarrow -500 = A(0) + c \Rightarrow c = -500$) so

$$s'(t) = At - 500$$

and distance $s(t) = \frac{A}{2}t^2 - 500t + 120$

$\text{(and } s(0) = 120 \Rightarrow 120 = \frac{A}{2}(0)^2 - 500(0) + c \Rightarrow c = 120\text{) so}$

$$s(t) = \frac{A}{2}t^2 - 500t + 120$$

Now

$$s'(t) = -180 \Rightarrow AT - 500 = -180 \Rightarrow AT = 320 \quad \text{and}$$

$$s(t) = 0 \Rightarrow \frac{A}{2}t^2 - 500t + 120 = 0 \quad \Rightarrow \frac{320}{1} \Rightarrow$$

$$\frac{1}{2} \cdot \frac{320}{1} \cdot t^2 - 500t + 120 = 0 \Rightarrow -340t = -120 \Rightarrow \boxed{T = 0.35 \text{ hr.}}$$

and $A = 90.7 \text{ mph/hr.}$

\[190: 26\]

a.) $s(0) = 10 \text{ ft, } s'(0) = 8 \text{ ft/sec, } s''(t) = -8 \text{ ft/sec}^2$

\(\Rightarrow\) $s'(t) = -8t + c$ \(\Rightarrow\) $s'(0) = 8 \Rightarrow c = 8$

\(\Rightarrow\) $s'(t) = -8t + 8 \Rightarrow s(t) = -4t^2 + 8t + C$ \(\Rightarrow\)

$C = 10$ \(\Rightarrow\) $s(t) = -4t^2 + 8t + 10$.

b.) $s(t)$ is largest when $s'(t) = 0$

\(\Rightarrow\) $-8t + 8 = 0 \Rightarrow t = 1 \text{ sec.} \Rightarrow \boxed{s(1) = 14 \text{ ft.}}$
\[ 190:29 \]

Let \( s(t) \) be distance traveled (\text{feet}) in \( t \) seconds. Then

\[
\begin{align*}
    s'(0) &= 0 \quad \text{ft./sec.,} \\
    s'(15) &= 88 \quad \text{ft./sec.,} \\
    s(0) &= 0 \quad \text{ft.} \\
\end{align*}
\]

\( s''(t) = A \rightarrow s'(t) = At + C \) (\( s'(0) = 0 \rightarrow C = 0 \) and \( s'(15) = 88 \rightarrow 88 = A(15) \rightarrow A = \frac{88}{15} \quad \text{ft./sec.}^2 \)) so

\[
    s'(t) = \frac{88}{15} t 
\]

and \( s(t) = \frac{44}{15} t^2 + C \) (\( s(0) = 0 \rightarrow C = 0 \))

\[
    s(15) = 660 \quad \text{ft.}
\]

\[ 190:30 \]

Assume a ball is thrown upward at \( v_0 \) \( \text{ft./sec.} \), then

\[
\begin{align*}
    s'' &= -32 \\
    s' &= -32t + C \quad (t = 0, s' = v_0) \rightarrow C = v_0 \rightarrow \\
    s' &= -32t + v_0 \rightarrow \\
    s &= -16t^2 + v_0 t + C \quad (t = 0, s = 0) \rightarrow C = 0 \rightarrow \\
    s &= -16t^2 + v_0 t
\end{align*}
\]

**Highest Point:** \( s' = 0 \rightarrow -32t + v_0 = 0 \rightarrow t = \frac{v_0}{32} \) seconds;

**Strike Ground:** \( s = 0 \rightarrow -16t^2 + v_0 t = 0 \rightarrow \\
    t(-16t + v_0) = 0 \rightarrow -16t + v_0 = 0 \rightarrow t = \frac{v_0}{16} \) sec.;

\[
\begin{align*}
    t = 0 : \quad &s' = v_0, \quad \text{speed} = |v_0| = v_0 \\
    t = \frac{v_0}{16} : \quad &s' = -v_0, \quad \text{speed} = |-v_0| = v_0
\end{align*}
\]
1.) A ball is thrown straight up from the top of a 128-ft. high building at 24 ft./sec.
   a.) In how many seconds will the ball reach its highest point? How high is the ball above the ground at this point?
   b.) In how many seconds will the ball strike the ground?
   c.) What is the ball’s velocity as it strikes the ground?

2.) A ball is thrown straight down from the top of a 128-ft. high building at 32 ft./sec.
   a.) In how many seconds will the ball strike the ground?
   b.) What is the ball’s velocity as it strikes the ground?

3.) A ball is thrown horizontally from the top of a 128-ft. high building at 100 ft./sec.
   a.) In how many seconds will the ball strike the ground?
   b.) What is the ball’s velocity as it strikes the ground?
   c.) How far from the base of the building will the ball hit the ground?

4.) A car increases speed with constant acceleration from 0 miles per hour to 60 miles per hour in 15 seconds. How long does it take the car to go from 60 miles per hour to 100 miles per hour at the same constant acceleration? What is the car’s acceleration? How far does the car travel as it goes from 0 and 100 miles per hour?

5.) A car traveling at 40 ft./sec. applies the brakes and begins a constant deceleration. If the car comes to rest in 100 feet after it applies the brakes, how long does it take to come to a complete stop? What is the car’s deceleration?
Worksheet 3 Solutions

1.) \( s'' = -32 \Rightarrow s' = -32t + c \) \((t=0, s' = 24)\)

\[ 24 = 0 + c \Rightarrow c = 24 \]

\[ s' = -32t + 24 \]

\[ s = -16t^2 + 24t + c \] \((t=0, s=128)\)

\[ 128 = 0 + c \Rightarrow c = 128 \]

\[ s = -16t^2 + 24t + 128 \]

a.) highest point: \( s' = 0 \Rightarrow -32t + 24 = 0 \Rightarrow t = \frac{24}{32} = \frac{3}{4} \text{ sec and } s = 137 \text{ ft.} \)

b.) strike ground: \( s = 0 \Rightarrow 0 = -16t^2 + 24t + 128 = -8(2t^2 - 3t - 16) \Rightarrow t = \frac{3 \pm \sqrt{9 + 128}}{4} = \frac{3 + \sqrt{137}}{4} \approx 3.68 \text{ sec.} \)

c.) \( t = 3.68 \text{ sec} \Rightarrow s' = -93.6 \text{ ft./sec.} \)

2.) \( s'' = -32 \Rightarrow s' = -32t + c \) \((t=0, s' = -32)\)

\[ -32 = 0 + c \Rightarrow c = -32 \]

\[ s' = -32t - 32 \]

\[ s = -16t^2 - 32t + c \] \((t=0, s=128)\)

\[ 128 = 0 + c \Rightarrow c = 128 \]

\[ s = -16t^2 - 32t + 128 \]
a.) Strike ground: \( s = 0 \rightarrow \)
0 = \(-16t^2 + 32t + 128 = -16(t^2 + 2t - 8) \rightarrow \)
0 = \(-16(t + 4)(t - 2) \rightarrow t = 2 \text{ sec.} \)

b.) \( t = 2 \text{ sec.} \rightarrow s' = -96 \text{ ft./sec.} \)

3.)
\[
\begin{align*}
S'' &= -32 \\
S' &= -32t + c \quad (t = 0, S' = 0 \Rightarrow c = 0) \\
o &= 0 + c \rightarrow c = 0 \\
S' &= -32t \\
S &= -16t^2 + c \quad (t = 0, S = 128) \\
128 &= 0 + c \rightarrow c = 128 \\
S &= -16t^2 + 128 \\
\end{align*}
\]

a.) Strike ground: \( s = 0 \rightarrow 0 = -16t^2 + 128 \rightarrow \)
t = \( \sqrt{\frac{128}{16}} = \sqrt{8} \approx 2.83 \text{ sec.} \)

b.) \( t = 2.83 \text{ sec.} \rightarrow s' = -90.5 \text{ ft./sec.} \)

c.) As the ball drops for 2.83 seconds, its horizontal component of velocity remains constant at 100 ft./sec. Thus, the ball lands \((100)(2.83) = 283 \text{ ft. from the base of the building.}\)
Let $s = s(t)$ be distance (ft.) traveled by car after $t$ seconds.

4.) \[
\frac{60 \text{ mi.}}{\text{hr.}} = \left(\frac{60 \text{ mi.}}{\text{hr.}}\right)\left(\frac{5280 \text{ ft.}}{\text{mi.}}\right)\left(\frac{1 \text{ hr.}}{3600 \text{ sec.}}\right) = 88 \text{ ft./sec.}
\]

and

\[
\frac{100 \text{ mi.}}{\text{hr.}} \approx 146.7 \text{ ft./sec.}
\]

\[t=0 \text{ sec.} \quad t=15 \]

\[s=0 \text{ ft.} \quad s = 88 \text{ ft./sec.}
\]

\[s'=0 \text{ ft./sec.} \]

Assume acceleration is

\[s'' = k \text{ ft./sec.}^2\]

then velocity is

\[s' = kt + c \quad s'(0) = 0 \rightarrow 0 = 0 + c \rightarrow c = 0 \quad s'(15) = 15k = 88 \rightarrow k = \frac{88}{15}
\]

so car's acceleration is

\[s'' = \frac{88}{15} \approx 5.87 \text{ ft./sec.}^2\]

then

\[s' = \frac{88}{15} t \quad \text{so distance}\]

\[s = \frac{44}{15} t^2 + c \quad s(0) = 0 \rightarrow 0 = 0 + c \rightarrow c = 0 \rightarrow s = \frac{44}{15} t^2
\]

if $s' = 100 \text{ mph} \approx 146.7 \text{ ft./sec.}$

then

\[t = \frac{88}{15} \approx 146.7 \rightarrow t = 25 \text{ sec.}
\]

so it takes $25 - 15 = 10 \text{ sec.}$

to go from 60 mph to 100 mph.
\[ s(25) = \frac{44}{15} (25)^2 \approx 1833 \text{ ft}. \]

traveled from \(0\) to \(100\) mph.

5.) Let \( s = s(t) \) be distance (feet) traveled \( t \) seconds after brakes are applied;

\[
\begin{array}{c}
\text{t=0 sec.} \\
\text{s=0 ft.} \\
\text{s'}=40 \text{ ft./sec.}
\end{array}
\quad
\begin{array}{c}
\text{t=T sec.} \\
\text{s=100 ft.} \\
\text{s'}=0 \text{ ft./sec.}
\end{array}
\]

Assume deceleration is
\[ s'' = k \text{ ft./sec.}^2 \]
then velocity \[ s' = kt + c \]
\[ s'(0) = 40 \rightarrow 40 = 0 + c \rightarrow c = 40 \rightarrow s' = kt + 40 \]
\[ s'(T) = 0 \rightarrow 0 = kT + 40 \]
so distance \[ s = \frac{k}{2} t^2 + 40t + c \]
\[ s(0) = 0 \rightarrow 0 = 0 + c \rightarrow c = 0 \]
\[ s(T) = 100 \rightarrow 100 = \frac{k}{2} T^2 + 40T \]

system of equations: \[ kT = -40 \]
so \[ 100 = \frac{1}{2} (kT)T + 40T \]
\[ = \frac{1}{2} (-40)T + 40T = 20T \]
$T = 5 \text{ sec.}$ to stop the car;

and $kT = -40 \rightarrow 5k = -40 \rightarrow$
car's deceleration is

$5'' = k = -8 \text{ ft/sec}^2$