

Math 21A

Kouba

The Derivative of Sin x

FACTS: 1.) $\lim_{w \rightarrow 0} \frac{\sin w}{w} = 1$

2.) $\lim_{w \rightarrow 0} \frac{\cos w - 1}{w} = 0$

3.) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

Let $f(x) = \sin x$. Then $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cdot \cos h + \cos x \cdot \sin h - \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cdot (\cos h - 1) + \cos x \cdot \sin h}{h}$$

$$= \lim_{h \rightarrow 0} \left\{ \sin x \cdot \left(\frac{\cos h - 1}{h} \right) + \cos x \cdot \left(\frac{\sin h}{h} \right) \right\}$$

$$= \sin x \cdot (0) + \cos x \cdot (1)$$

$$= \cos x$$

i.e.,

$$D\{\sin x\} = \cos x$$