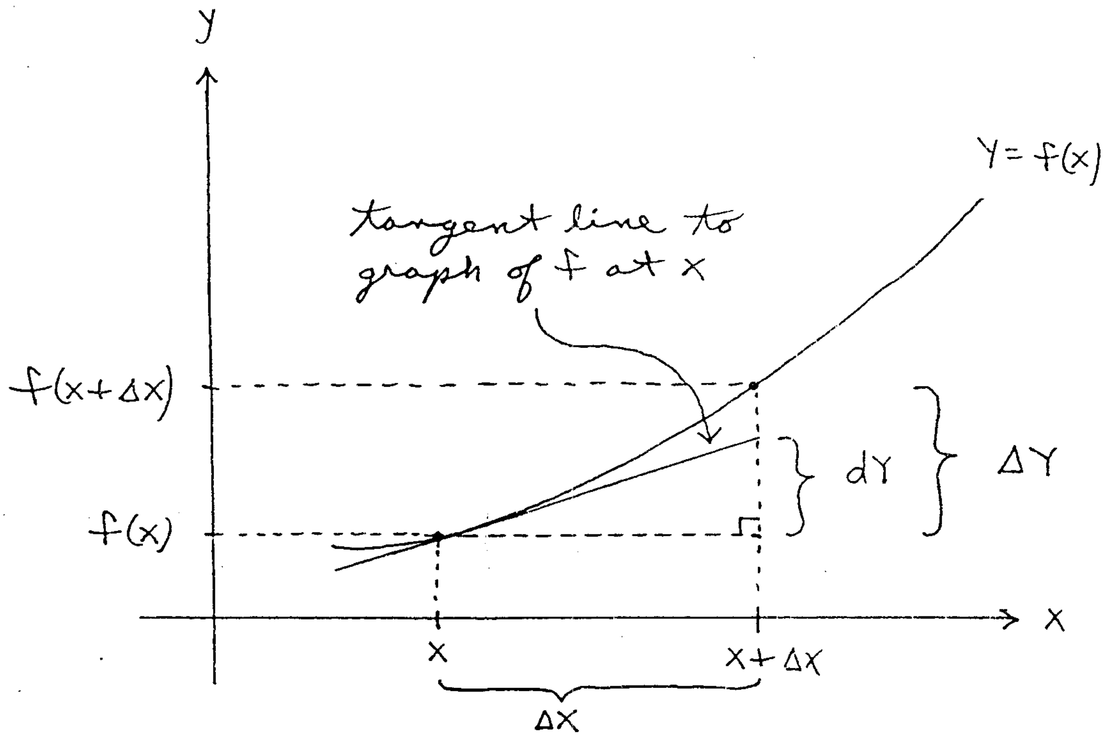
Math 21A
Kouba
The Differential



Define the exact change in y to be

$$\boxed{\Delta Y = f(x+\Delta x) - f(x)}\ .$$

Define dY to be the height of the right triangle in the diagram. Then

$$\boxed{\Delta Y \approx dY} \quad \text{for} \quad \Delta x \ \text{small}.$$

In addition, the slope of the tangent line to the graph of f at x is

$$f'(x) = \frac{\text{rise}}{\text{run}} = \frac{dY}{\Delta x} \quad \text{so that}$$

$$\boxed{dY = f'(x) \cdot \Delta x}\ ;$$

dY is called the differential of Y.