

Math 21A

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More Examples Using Differentials

Example 1: For small h , show that $\sqrt{4+3h^2} \approx 2 + \frac{3}{4}h^2$ using differentials.

Solution: Let $f(x) = \sqrt{x}$ and assume that $x: 4 \rightarrow 4+3h^2$. Then $\Delta x = 3h^2$ and $f'(x) = \frac{1}{2\sqrt{x}}$. Since Δx is small (because h is small) $\Delta f \approx df \rightarrow f(4+3h^2) - f(4) \approx f'(4) \cdot \Delta x$
 $\rightarrow \sqrt{4+3h^2} - \sqrt{4} \approx \frac{1}{2\sqrt{4}} \cdot 3h^2$
 $\rightarrow \sqrt{4+3h^2} - 2 \approx \frac{3}{4}h^2$
 $\rightarrow \sqrt{4+3h^2} \approx 2 + \frac{3}{4}h^2$.

Example 2: If the radius of a circle is measured with an absolute percentage error of at most 3%, use differentials to estimate the maximum absolute percentage error in computing the circle's a.) circumference. b.) area.

Solution: assume that $\frac{|\Delta r|}{r} \leq 3\%$.

a.) $C = 2\pi r$, $C' = 2\pi$, find $\frac{|\Delta C|}{C}$:

$$\frac{|\Delta C|}{C} \approx \frac{|dC|}{C} = \frac{|C' \cdot \Delta r|}{C} = \frac{|2\pi \cdot \Delta r|}{2\pi r} = \frac{|\Delta r|}{r} \leq 3\%$$

b.) $A = \pi r^2$, $A' = 2\pi r$, find $\frac{|\Delta A|}{A}$:

$$\frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A' \cdot \Delta r|}{A} = \frac{|2\pi r \cdot \Delta r|}{\pi r^2} = 2 \frac{|\Delta r|}{r} \leq 2(3\%) = 6\%$$