Math 21A

Kouba

Discussion Sheet 5
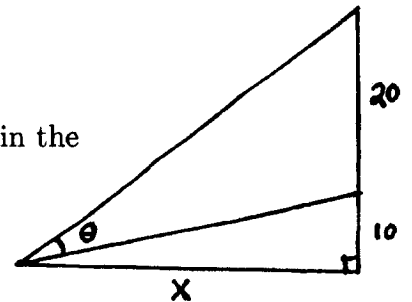
1.) Use any method to differentiate the following functions. You need not simplify answers.

a.) $y = 7\sec 3x$

b.) $f(x) = \dfrac{x\sin x}{1 + \tan x}$

c.) $f(x) = \left(\dfrac{3x + 7}{7x - 9}\right)^{50}$

d.) $g(x) = x^3 \cos x^2$

e.) $f(x) = \sqrt{1 + \sqrt{2 + \sqrt{3 - x}}}$

f.) $g(x) = \sin 2x \cdot \cos^2 x$

g.) $y = \cot^5\left(\sin^3\left(10x^5\right)\right)$

2.) Derive a formula for the measure of angle $\theta$ as a function of $x$ in the given diagram.

3.) For each of the following functions solve $f'(x) = 0$ for $x$ and $f''(x) = 0$ for $x$.

a.) $f(x) = x(x - 5)^4$

b.) $f(x) = \dfrac{x^3}{x - 2}$

c.) $f(x) = (x^2 - 4)^3$

4.) Find an equation of the line tangent to the graph of $y = \dfrac{x}{x^2 + 3}$ at $x = -1$ .

5.) Find an equation of the line perpendicular to (normal to) the graph of $y = \dfrac{\cos x}{\sin x + \cos 3x}$ at $x = \pi/6$ .

6.) Let $f(x) = \dfrac{2 - x}{x + 3}$ .

   a.) Find the Average Rate of Change (ARC) of $f$ on intervals i.) $[-2, 1]$  ii.) $[-6, -4]$
   b.) Find the Instantaneous Rate of Change (IRC) of $f$ for  i.) $x = 1$  ii.) $x = -4$

7.) The weight (pounds) $w(t)$ of a buffalo calf at time $t$ during its first eight months of life is given in the graph on the following page.

   a.) Find the ARC of $w$ on intervals  i.) $[0, 2]$  ii.) $[5, 8]$  iii.) $[0, 3.6]$ .
   b.) Find the IRC of $w$ for  i.) $t = 1$  ii.) $t = 2$  iii.) $t = 6$ .
   c.) Estimate the exact time when the IRC of $w$ is LARGEST. What is the IRC at this time ?

   d.) Estimate the exact time when the IRC of $w$ is SMALLEST. What is the IRC at this time ?

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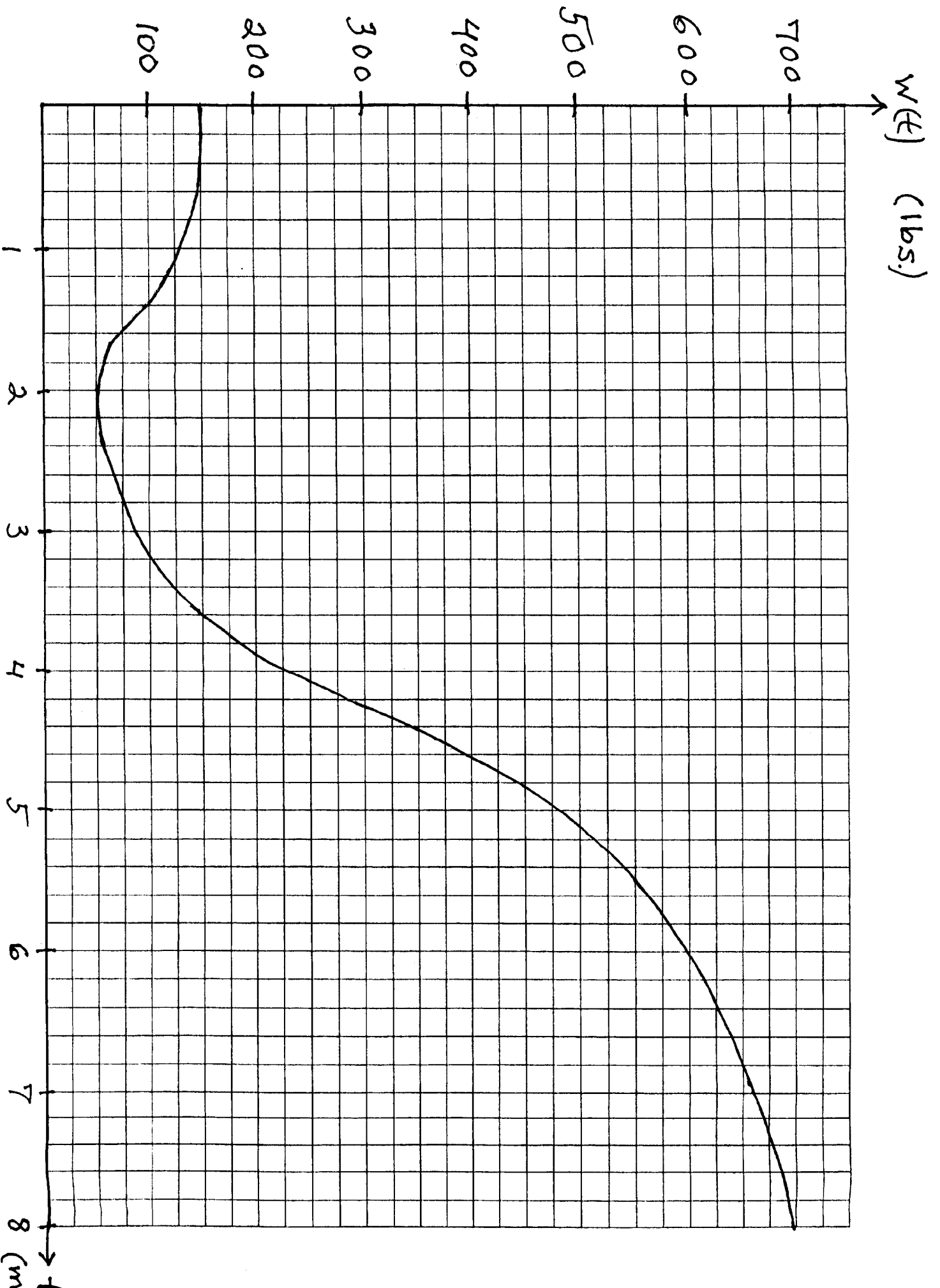
8.) Find all points $(x, y)$ which are equidistant from the three given points $(0, 0)$, $(4, 0)$, and $(3, 2)$ .

9.) Derive equations of all lines which are tangent to the graph of $y = -7 - x^2$ and passing through the point $(3, 0)$ (This point is NOT on the graph.).

10.) A spaceship is traveling (left to right) along the curve $y = 3\cos x$ . An object is released from the spaceship at $x = \pi/3$ and travels along a line tangent to the graph of $y = 3\cos x$ towards the x-axis.

    a.) At what point $x$ will the object strike the x-axis ?
    b.) At what angle $\theta$ will the object strike the x-axis ?

11.) A white rat is crawling along the the graph of $y = x^2(x - 3)$. It's position at time $t$ minutes is given by $(x, y) = (x(t), y(t))$. If the rat's x-coordinate is changing at the constant rate of 3 cm./min., at what rate is the rat's y-coordinate changing when i.) $x = 2$ ii.) $x = 6$ ?

12.) The position $x(t)$ (feet) of an object on the x-axis at time t (seconds) is given by $x(t) = t^2(2 - t)^3$. Find the object's

    a.) position
    b.) velocity
    c.) acceleration
    d.) jerk

for i.) $t = 1$ ii.) $t = 3$ .

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The following problem is for recreational purposes only.

13.) Assume that you have three boxes labeled and filled with fruit. One box contains APPLES only. One box contains ORANGES only. One box contains APPLES and OR-ANGES. Unfortunately, ALL of the boxes are labeled incorrectly. Explain how to correctly relabel all of the boxes by (without peeking into any box) selecting exactly one fruit from exactly one box.

"Gravity is not responsible for people falling in love." – Albert Einstein

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