Math 21A
Kouba
Discussion Sheet 6

1.) Assume that y is a function of x. Compute $y' = \dfrac{dy}{dx}$ and $y'' = \dfrac{d^2y}{dx^2}$ (You need not simplify y''.) for each equation.

a.) $y = x + x^3$

b.) $x = y + y^3$

c.) $y = x^2 \sin x$

d.) $y = x^2 \sin y$

e.) $x^2 + y^3 = xy$

f.) $(x - y)^3 = x^2 - y^2$

g.) $\sin(3y) + \tan^2 y = \cos x$

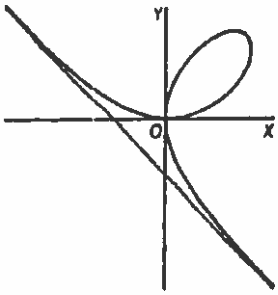
h.) $xe^y + ye^x = 1$

i.) $\ln y + \ln(y + 3x) = 2$

j.) $e^{xy^2} = 2^y + \sec x$

2.) Use implicit differentiation to find the largest y-value in the "loop" of the Folium of Descartes, which is given by the equation $x^3 + y^3 - 3xy = 0$ (See diagram below.).

Folium of Descartes



3.) Find the values of y' and y'' for $xy^2 + y = 2$ at $x = 0$ and at $x = 1$.

4.) Let $f(x) = x^3 \ln x$. Solve $f'(x) = 0$ and $f''(x) = 0$ for x and set up a sign chart for each.

5.) Let $f(x) = x^2 \cdot e^{-x}$. Solve $f'(x) = 0$ and $f''(x) = 0$ for x and set up a sign chart for each.

6.) Find $y' = \dfrac{dy}{dx}$ as simply as possible. Do not simplify your answers.

a.) $y = \ln(5x+7)$ b.) $y = \ln(\ln(\ln(\sin x)))$ c.) $y = \ln\left(\dfrac{x \cdot \sin(3x) \cdot (x-2)^4}{(x+1)^5 \cdot \cos(2x)}\right)$

d.) $y = x^{\ln 7} + \log_7 x$ e.) $y = x^3 \cdot (x-1)^5 \cdot (2x+3)^7 \cdot \tan^3 x$

f.) $y = (2x)^{x+3}$ g.) $y = x^{y^2}$ h.) $y = x^{(y^{\sin x})}$ i.) $y = \log_5(\log_4(\log_3 2x))$

7.) Find $y' = \dfrac{dy}{dx}$. Do not simplify your answers.

a.) $y = 7 \cdot e^{5x-4}$ b.) $y = e^{x^2} \cdot \tan(3x)$ c.) $y = \dfrac{2^x + 4^x}{3^x + 5^x}$ d.) $y = 4^{3^{x^5}}$

8.) Find all horizontal asymptotes for the function in 7.)c.).

9.) Differentiate.

a.) $y = \arctan(3x)$ b.) $y = x \cdot \arcsin(e^{2x})$ c.) $y = \arccos(\sin x^2)$

d.) $y = \cos^2((\arctan x)^3)$ e.) $y = \dfrac{\arccos x - \arctan x}{\arctan x - \arcsin x}$

f.) $y = \arcsin\left(\dfrac{x-1}{x+1}\right) + 2\arctan\sqrt{x}$ (Simplify your answer as much as possible.)

10.) Show that the following function

a.) $f(x) = 3 - 2x + \sin 2x$ is one-to-one using a derivative sign chart.

b.) $f(x) = \dfrac{1-2x}{5+3x}$ is one-to-one using the algebraic test.

11.) The function $f(x) = (5x^{1/3} - 9)^5 + 2$ is one-to-one and has an inverse function, $f^{-1}(x)$, with $f(8) = 3$ so that $f^{-1}(3) = 8$.

a.) Find the inverse function, f^{-1}.

b.) Find the derivative $\dfrac{d}{dx} f^{-1}(3)$

i.) directly. ii.) using $\dfrac{d}{dx} f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))}$.

12.) Show that the function $f(x) = x^3 + 2x + 1$ is one-to-one. Then find the derivative $\dfrac{d}{dx} f^{-1}(4)$.

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The following problem is for recreational purposes only.

13.) A horse is tethered by a rope to the corner of a small shed with a square 10 ft. by 10 ft. floor. If the rope is 40 feet long, sketch the shape of the horse's grazing area. How close can you plant flowers to the shed and keep the horse from eating them ?