Math 21A
Kouba
Discussion Sheet 7

1.) The volume V of a sphere is changing at the rate of π ft.^{3}/min. At what rate is the sphere's surface area S changing when

    a.) S = 4π ft.^{2} ?   b.) S = 36π ft.^{2} ?

2.) Car B is 30 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. At what rate is the distance between the cars changing after t = 1/5 hr. ? t = 1/3 hr. ?

3.) A conical tank (point down) has height 10 ft. and base radius 8 ft. Water begins flowing into the tank at the rate of π ft.^{3}/sec. At what rate is the depth h of the water changing

    a.) when $h = 1$ ft. ?   b.) when $h = 9$ ft. ?

4.) An open hemispherical tank has radius 13 meters. Water begins flowing into the tank in such a way that the depth of water h changes at the rate of 3 m./min. At what rate is the top circular surface area of the water changing

    a.) when $h = 1$ m. ?   b.) when $h = 8$ m. ?

5.) Consider the function $f(x) = \sqrt{x+3}$. Set up a table of values for Δx, Δf, and df for the given values of x.

    a.) $x : 1 \to 3$ b.) $x : 1 \to 1.5$ c.) $x : 1 \to 1.2$ d.) $x : 1 \to 1.1$

6.) Use a differential to estimate the value of each number. Compare your differential estimate with the calculator value of each number.

    a.) $\sqrt{27}$ (Use 25 as a convenient, nearby x-value.)   b.) $\sqrt{27}$ (Use 36 as a convenient, nearby x-value.)

    c.) $12^{1/3}$ d.) $(9900)^{1/4}$ e.) $\ln 1.2$ f.) $e^{-0.1}$

7.) The radius of a sphere is measured with absolute percentage error of at most 4%. Use differentials to estimate the maximum absolute percentage error in computing the sphere's

    a.) surface area.   b.) volume.

8.) The surface area of a cube of side length x is measured with absolute percentage error of at most 12%. Use differentials to estimate the maximum absolute percentage error in computing the cube's

    a.) side length x.   b.) volume.

9.) Find the linearization (tangent line) for each function at the given value of x.

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a.) $f(x) = \sqrt{x+4}$ at $x = 0$

b.) $f(x) = \sqrt{x+4}$ at $x = -1$

c.) $f(x) = 3 + x \sin x$ at $x = 0$

d.) $f(x) = x^3 - 2x + \cos x$ at $x = 0$

e.) $f(x) = \dfrac{4e^x}{e^x + 1}$ at $x = 0$

f.) $f(x) = x^x$ at $x = 1$

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The following problem is for recreational purposes only.

10.) Find a hidden pattern and determine the next number in the sequence :

$$0, 1, 3, 7, 14, 25, 41, 63, \cdots$$

11.) Try to figure out what the value of this "continued fraction" is :

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}$$