Math 21A
Kouba
An Extreme Example

The following function, which requires some knowledge of infinite series, represents a function which is *continuous* for all values of x, but *differentiable at no values of x.* The source is Principles of Mathematical Analysis, by Walter Rudin, McGraw-Hill, 1964, page 141.

7.18. Theorem. *There exists a real continuous function on the real line which is nowhere differentiable.*

Proof: Define

(44) $$\phi(x) = \begin{cases} x & (0 \le x \le 1), \\ 2 - x & (1 \le x \le 2) \end{cases}$$

and extend the definition of $\phi(x)$ to all real x by requiring that

$$\phi(x + 2) = \phi(x).$$

Then ϕ is continuous on R^1. Define

(45) $$f(x) = \sum_{n=0}^{\infty} \left(\tfrac{3}{4}\right)^n \phi(4^n x).$$