Section 3.8

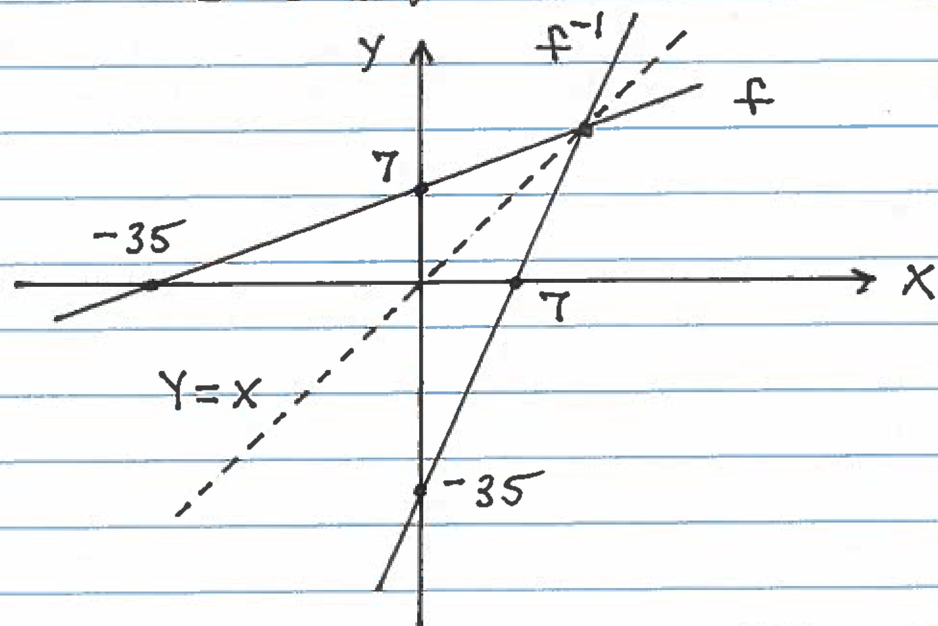
2.) $f(x) = \frac{1}{5}x + 7$, $a = -1$, then

a.) $Y = \frac{1}{5}X + 7 \rightarrow X = \frac{1}{5}Y + 7 \rightarrow$

$\frac{1}{5}Y = X - 7 \rightarrow Y = 5X - 35 \rightarrow$

inverse $f^{-1}(x) = 5X - 35$

b.)



c.) $f(-1) = \frac{1}{5}(-1) + 7 = \frac{-1}{5} + \frac{35}{5} = \frac{34}{5}$

$\frac{df}{dx} = \frac{1}{5}$, $\frac{df^{-1}}{dx} = 5 \rightarrow$

$$\frac{df}{dx} = \frac{1}{5} = \frac{1}{\frac{df^{-1}}{dx}}$$

7.) $f(x) = x^3 - 3x^2 - 1$ (for $x \geq 2$) $\xrightarrow{D}$

$f'(x) = 3x^2 - 6x$, $f(3) = -1$ so $f^{-1}(-1) = 3$:

$$\frac{df^{-1}}{dx}(-1) = \frac{1}{f'(f^{-1}(-1))} =$$

$$= \frac{1}{f'(3)}$$

$$= \frac{1}{3(3)^2 - 6(3)} = \frac{1}{9}$$

8.) $f(x) = x^2 - 4x - 5$ (for $x > 2$) $\xrightarrow{D}$

$f'(x) = 2x - 4$, $f(5) = 0$ so $f^{-1}(0) = 5$:

$$\frac{df^{-1}}{dx}(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(5)}$$

$$= \frac{1}{2(5) - 4} = \frac{1}{6}$$

9.) $y = f(x)$ and $f(2) = 4$ so $f^{-1}(4) = 2$, and slope $f'(2) = 1/3$. Then

$$\frac{df^{-1}}{dx}(4) = \frac{1}{f'(f^{-1}(4))}$$

$$= \frac{1}{f'(2)}$$

$$= \frac{1}{1/3} = 3$$

11.) $Y = \ln 3x \xrightarrow{D} Y' = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

16.) $Y = \ln\left(\frac{10}{x}\right) = \ln 10 - \ln x \xrightarrow{D}$

$Y' = \frac{-1}{x}$

19.) $Y = \ln x^3 = 3 \cdot \ln x \xrightarrow{D}$

$Y' = 3 \cdot \frac{1}{x}$

20.) $Y = (\ln x)^3 \xrightarrow{D} Y' = 3(\ln x)^2 \cdot \frac{1}{x}$

22.) $Y = t\,(\ln t)^{1/2} \xrightarrow{D}$

$Y' = t \cdot \frac{1}{2}(\ln t)^{-1/2} \cdot \left(\frac{1}{t}\right) + (1) \cdot (\ln t)^{1/2}$

25.) $Y = \frac{\ln t}{t} \xrightarrow{D} y' = \frac{t \cdot \left(\frac{1}{t}\right) - \ln t \cdot (1)}{t^2}$

28.) $Y = \frac{x \ln x}{1 + \ln x} \xrightarrow{D}$

$$Y' = \frac{(1+\ln x) \cdot \left(x \cdot \frac{1}{x} + (1)\ln x\right) - x \ln x \cdot \left(\frac{1}{x}\right)}{(1+\ln x)^2}$$

30.) $Y = \ln(\ln(\ln x)) \xrightarrow{D}$

$Y' = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$

32.) $Y = \ln(\sec\theta + \tan\theta) \xrightarrow{D}$

$Y' = \frac{1}{\sec\theta + \tan\theta} \cdot (\sec\theta\tan\theta + \sec^2\theta)$

37.) $Y = \ln(\sec(\ln\theta)) \xrightarrow{D}$

$$Y' = \frac{1}{\sec(\ln\theta)} \cdot \sec(\ln\theta)\tan(\ln\theta) \cdot \frac{1}{\theta}$$

39.) $Y = \ln \frac{(x^2+1)^5}{(1-x)^{1/2}} = \ln(x^2+1)^5 - \ln(1-x)^{1/2}$

$= 5\ln(x^2+1) - \frac{1}{2}\ln(1-x) \xrightarrow{D}$

$Y' = 5 \cdot \frac{1}{x^2+1} \cdot 2x - \frac{1}{2} \cdot \frac{1}{1-x} \cdot (-1)$

40.) $Y = \ln\left(\frac{(x+1)^5}{(x+2)^{20}}\right)^{1/2} = \ln \frac{(x+1)^{5/2}}{(x+2)^{10}}$

$= \ln(x+1)^{5/2} - \ln(x+2)^{10} = \frac{5}{2}\ln(x+1) - 10\ln(x+2) \xrightarrow{D}$

$Y' = \frac{5}{2} \cdot \frac{1}{x+1} - 10 \cdot \frac{1}{x+2}$

51.) $Y = \frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} \longrightarrow \ln Y = \ln \frac{x \cdot (x^2+1)^{1/2}}{(x+1)^{2/3}}$

$\longrightarrow \ln Y = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1) \xrightarrow{D}$

$\frac{1}{Y}Y' = \frac{1}{x} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}x}{x^2+1} - \frac{2}{3} \cdot \frac{1}{x+1} \longrightarrow$

$Y' = \frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} \cdot \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3} \cdot \frac{1}{x+1}\right]$

54.) $Y = \left(\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}\right)^{1/3} \longrightarrow$

$\ln Y = \ln\left(\frac{x^{1/3} \cdot (x+1)^{1/3} \cdot (x-2)^{1/3}}{(x^2+1)^{1/3}(2x+3)^{1/3}}\right) \longrightarrow$

$\ln Y = \frac{1}{3}\ln x + \frac{1}{3}\ln(x+1) + \frac{1}{3}\ln(x-2)$

$- \frac{1}{3}\ln(x^2+1) - \frac{1}{3}\ln(2x+3) \xrightarrow{D}$

$$\frac{1}{Y}Y' = \frac{1}{3}\cdot\frac{1}{X} + \frac{1}{3}\cdot\frac{1}{X+1} + \frac{1}{3}\cdot\frac{1}{X-2}$$
$$-\frac{1}{3}\cdot\frac{2X}{X^2+1} - \frac{1}{3}\cdot\frac{2}{2X+3} \longrightarrow$$
$$Y' = \left(\frac{X(X+1)(X-2)}{(X^2+1)(2X+3)}\right)^{1/3}\cdot\left[\frac{1}{3X} + \frac{1}{3(X+1)} + \frac{1}{3(X-2)}\right.$$
$$\left.-\frac{2X}{3(X^2+1)} - \frac{2}{3(2X+3)}\right]$$

59.) $$Y = \ln\left(\frac{e^\theta}{1+e^\theta}\right) = \ln e^\theta - \ln(1+e^\theta)$$
$$= \theta - \ln(1+e^\theta) \xrightarrow{D}$$
$$Y' = 1 - \frac{1}{1+e^\theta}\cdot e^\theta$$

61.) $$Y = e^{(\cos t + \ln t)} \xrightarrow{D}$$
$$y' = e^{(\cos t + \ln t)}\cdot\left(-\sin t + \frac{1}{t}\right)$$

65.) $$X^Y = Y^X \longrightarrow \ln X^Y = \ln Y^X \longrightarrow$$
$$Y\cdot\ln X = X\cdot\ln Y \xrightarrow{D}$$
$$Y\cdot\frac{1}{X} + Y'\cdot\ln X = X\cdot\frac{1}{Y}Y' + (1)\cdot\ln Y \longrightarrow$$
$$Y'\cdot\ln X - \frac{X}{Y}Y' = \ln Y - \frac{Y}{X} \longrightarrow$$
$$Y'\left(\ln X - \frac{X}{Y}\right) = \ln Y - \frac{Y}{X} \longrightarrow$$
$$Y' = \frac{\ln Y - Y/X}{\ln X - X/Y}.$$

66.) $\tan y = e^x + \ln x \xrightarrow{D}$

$\sec^2 y \cdot y' = e^x + 1/x \longrightarrow$

$$y' = \frac{e^x + 1/x}{\sec^2 y}$$

68.) $y = 3^{-x} \xrightarrow{D} y' = 3^{-x} \cdot (-1) \cdot \ln 3$

71.) $y = x^{\pi} \xrightarrow{D} y' = \pi x^{\pi - 1}$

74.) $y = \log_3 (1 + \theta \ln 3) \xrightarrow{D}$

$$y' = \frac{1}{1+\theta \ln 3} \cdot \cancel{\ln 3} \cdot \frac{1}{\cancel{\ln 3}} = \frac{1}{1+\theta \ln 3}$$

82.) $y = \log_7 \left(\frac{\sin\theta \cdot \cos\theta}{e^{\theta} \cdot 2^{\theta}} \right)$

$= \log_7(\sin\theta) + \log_7(\cos\theta) - \log_7 e^{\theta} - \log_7 2^{\theta}$

$= \log_7(\sin\theta) + \log_7(\cos\theta) - \theta \cdot \log_7 e - \theta \log_7 2$

$\xrightarrow{D} y' = \frac{1}{\sin\theta} \cdot \cos\theta \cdot \frac{1}{\ln 7} + \frac{1}{\cos\theta} \cdot -\sin\theta \cdot \frac{1}{\ln 7}$

$- \log_7 e - \log_7 2$

85.) $y = 3^{\log_2 t} \xrightarrow{D} y' = 3^{\log_2 t} \cdot \ln 3 \cdot \frac{1}{t} \cdot \frac{1}{\ln 2}$

86.) $y = 3 \log_8 (\log_2 t) \xrightarrow{D}$

$$y' = 3 \cdot \frac{1}{\log_2 t} \cdot \frac{1}{\ln 8} \cdot \frac{1}{t} \cdot \frac{1}{\ln 2}$$

90.) $y = x^{x+1} \longrightarrow \ln y = \ln x^{x+1} = (x+1)\ln x \xrightarrow{D}$

$\frac{1}{y}\cdot y' = (x+1)\cdot\frac{1}{x} + (1)\cdot\ln x \longrightarrow$

$y' = x^{x+1}\cdot\left(\frac{x+1}{x} + \ln x\right)$

92.) $y = t^{\sqrt{t}} \longrightarrow \ln y = \ln t^{\sqrt{t}} = \sqrt{t}\cdot\ln t \xrightarrow{D}$

$\frac{1}{y}y' = \sqrt{t}\cdot\frac{1}{t} + \frac{1}{2}t^{-1/2}\cdot\ln t \longrightarrow$

$y' = t^{\sqrt{t}}\cdot\left(\frac{1}{\sqrt{t}} + \frac{1}{2}t^{-1/2}\cdot\ln t\right)$

93.) $y = (\sin x)^x \longrightarrow \ln y = \ln(\sin x)^x \longrightarrow$

$\ln y = x\cdot\ln(\sin x) \xrightarrow{D}$

$\frac{1}{y}y' = x\cdot\frac{1}{\sin x}\cdot\cos x + (1)\cdot\ln(\sin x) \longrightarrow$

$y' = (\sin x)^x\cdot[x\cot x + \ln(\sin x)]$

96.) $y = (\ln x)^{\ln x} \longrightarrow \ln y = \ln(\ln x)^{\ln x} \longrightarrow$

$\ln y = \ln x\cdot\ln(\ln x) \xrightarrow{D}$

$\frac{1}{y}y' = \cancel{\ln x}\cdot\frac{1}{\cancel{\ln x}}\cdot\frac{1}{x} + \frac{1}{x}\cdot\ln(\ln x) \longrightarrow$

$y' = (\ln x)^{\ln x}\cdot\left[\frac{1}{x} + \frac{\ln(\ln x)}{x}\right]$