Section 4.5

2.) $\lim_{x\to 0} \frac{\sin 5x}{x} \overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{5\cos 5x}{1}$

$= \frac{5\cos 0}{1} = 5(1) = 5$

4.) $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3} \overset{"\frac{0}{0}"}{=} \lim_{x\to 1} \frac{3x^2}{12x^2-1}$

$= \frac{3}{12-1} = \frac{3}{11}$

5.) $\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{-(-\sin x)}{2x}$

$\overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{\cos x}{2} = \frac{\cos 0}{2} = \frac{1}{2}$

6.) $\lim_{x\to \infty} \frac{2x^2+3x}{x^3+x+1} \overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to \infty} \frac{4x+3}{3x^2+1}$

$\overset{"\frac{\infty}{\infty}"}{=} \lim_{x\to \infty} \frac{4}{6x} = \frac{4}{\infty} = 0$

7.) $\lim_{x\to 2} \frac{x-2}{x^2-4} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 2} \frac{1}{2x} = \frac{1}{4}$

10.) $\lim_{t\to -1} \frac{3t^3+3}{4t^3-t+3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to -1} \frac{9t^2}{12t^2-1} = \frac{9}{11}$

13.) $\lim_{t\to 0} \frac{\sin t^2}{t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 0} \frac{\cos t^2 \cdot 2t}{1} = (1)(0) = 0$

14.) $\lim_{t\to 0} \frac{\sin 5t}{2t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 0} \frac{\cos 5t \cdot 5}{2} = \frac{(1)\cdot 5}{2} = \frac{5}{2}$

15.) $\lim_{x\to 0} \frac{8x^2}{\cos x - 1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{16x}{-\sin x}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{16}{-\cos x} = \frac{16}{-1} = -16$

18.) $\lim_{\theta\to -\frac{\pi}{3}} \frac{3\theta+\pi}{\sin(\theta+\frac{\pi}{3})} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{\theta\to -\frac{\pi}{3}} \frac{3}{\cos(\theta+\frac{\pi}{3})} = \frac{3}{1} = 3$

20.) $\lim_{x\to 1} \frac{x-1}{\ln x - \sin \pi x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{1}{\frac{1}{x} - \pi \cos \pi x}$

$= \frac{1}{1-\pi(-1)} = \frac{1}{1+\pi}$

21.) $\lim_{x\to 0} \frac{x^2}{\ln(\sec x)} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2x}{\frac{1}{\cancel{\sec x}} \cdot \cancel{\sec} x \tan x}$

$= \lim_{x\to 0} \frac{2x}{\tan x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2}{\sec^2 x} = \frac{2}{(1)^2} = 2$

24.) $\lim_{t\to 0} \frac{t \sin t}{1-\cos t} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t\to 0} \frac{t\cos t + \sin t}{\sin t}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{t \to 0} \frac{t \cdot -\sin t + \cos t + \cos t}{\cos t}$$

$$= \frac{0+1+1}{1} = 2$$

26.) $\displaystyle \lim_{x \to \frac{\pi}{2}^-} \left(\frac{\pi}{2} - x\right) \tan x = \text{"}0 \cdot \infty\text{"}$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\pi}{2} - x}{\cot x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to \frac{\pi}{2}^-} \frac{-1}{-\csc^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \sin^2 x = (1)^2 = 1$$

27.) $\displaystyle \lim_{\theta \to 0} \frac{3^{\sin\theta} - 1}{\theta} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{\theta \to 0} \frac{3^{\sin\theta} \cdot \cos\theta \cdot \ln 3}{1}$

$$= (1)(1) \ln 3 = \ln 3$$

29.) $\displaystyle \lim_{x \to 0} \frac{x \cdot 2^x}{2^x - 1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{x \cdot 2^x \ln 2 + 2^x}{2^x \ln 2}$

$$= \frac{0+1}{(1)\ln 2} = \frac{1}{\ln 2}$$

32.) $\displaystyle \lim_{x \to \infty} \frac{\log_2 x}{\log_3 (x+3)} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \to \infty} \frac{\frac{1}{x} \cdot \frac{1}{\ln 2}}{\frac{1}{x+3} \cdot \frac{1}{\ln 3}}$

$$= \lim_{x \to \infty} \frac{x+3}{x} \cdot \frac{\ln 3}{\ln 2} = \lim_{x \to \infty} \left(1 + \frac{3}{x}\right) \cdot \frac{\ln 3}{\ln 2}$$

$$= (1+0) \frac{\ln 3}{\ln 2} = \frac{\ln 3}{\ln 2}$$

34.) $\lim\limits_{x\to 0^+} \dfrac{\ln(e^x-1)}{\ln x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{e^x-1}\cdot e^x}{\frac{1}{x}}$

$= \lim\limits_{x\to 0^+} \dfrac{xe^x}{e^x-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{xe^x+e^x}{e^x} = \dfrac{1}{1} = 1$

35.) $\lim\limits_{Y\to 0} \dfrac{\sqrt{5Y+25}-5}{Y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{Y\to 0} \dfrac{\frac{1}{2}(5Y+25)^{-1/2}\cdot 5}{1}$

$= \lim\limits_{Y\to 0} \dfrac{5}{2\sqrt{5Y+25}} = \dfrac{5}{2(5)} = \dfrac{1}{2}$

37.) $\lim\limits_{x\to\infty} (\ln 2x - \ln(x+1)) = \text{"}\infty - \infty\text{"}$

$= \lim\limits_{x\to\infty} \ln\left(\dfrac{2x}{x+1}\right) = \ln\left(\lim\limits_{x\to\infty} \dfrac{2x}{x+1}\right)$

$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \ln\left(\lim\limits_{x\to\infty} \dfrac{2}{1}\right) = \ln 2$

38.) $\lim\limits_{x\to 0^+} (\ln x - \ln(\sin x)) = \text{"}\infty - \infty\text{"}$

$= \lim\limits_{x\to 0^+} \ln\left(\dfrac{x}{\sin x}\right) = \ln\left(\lim\limits_{x\to 0^+} \dfrac{x}{\sin x}\right)$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \ln\left(\lim\limits_{x\to 0^+} \dfrac{1}{\cos x}\right) = \ln\left(\dfrac{1}{\cos 0}\right) = \ln\left(\dfrac{1}{1}\right) = 0$

39.) $\lim\limits_{x\to 0^+} \dfrac{(\ln x)^2}{\ln(\sin x)} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{2\ln x\cdot 1/x}{\frac{1}{\sin x}\cdot\cos x}$

$$= \lim_{x\to 0^+} \frac{2\sin x \ln x}{x\cos x}$$

$$= \left(\lim_{x\to 0^+} \frac{2\sin x}{x}\right)\cdot\left(\lim_{x\to 0^+} \frac{\ln x}{\cos x}\right)$$

$$= \left(\lim_{x\to 0^+} \frac{2\cos x}{1}\right)\cdot\left(\frac{-\infty}{\cos 0}\right)$$

$$= 2\cancel{\cos 0}^{\,1}\cdot\left(\frac{-\infty}{1}\right) = -\infty$$

41.) $\displaystyle\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right) = \text{"}\infty-\infty\text{"}$

$$= \lim_{x\to 1^+} \frac{\ln x-(x-1)}{(x-1)\ln x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}-1}{(x-1)\cdot\frac{1}{x}+(1)\ln x}\cdot\frac{x}{x}$$

$$= \lim_{x\to 1^+} \frac{1-x}{x-1+x\ln x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1^+} \frac{-1}{1+x\cdot\frac{1}{x}+(1)\ln x}$$

$$= \lim_{x\to 1^+} \frac{-1}{1+1+\ln x} = \frac{-1}{2+0} = \frac{-1}{2}$$

42.) $\displaystyle\lim_{x\to 0^+}\left(\csc x-\cot x+\cos x\right)$

$$= \lim_{x\to 0^+}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}+\cos x\right) = \text{"}\infty-\infty\text{"}$$

$$= \lim_{x\to 0^+} \frac{1-\cos x}{\sin x} + \lim_{x\to 0^+} \cos x$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+} \frac{-(-\sin x)}{\cos x} + \cos 0$$

$$= \frac{\sin 0}{\cos 0} + 1 = \frac{0}{1} + 1 = 1$$

44.) $$\lim_{h\to 0} \frac{e^h - 1 - h}{h^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h\to 0} \frac{e^h - 1}{2h}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h\to 0} \frac{e^h}{2} = \frac{1}{2}$$

46.) $$\lim_{x\to\infty} \frac{x^2}{e^x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2}{e^x}$$

$$= \frac{2}{\infty} = 0$$

47.) $$\lim_{x\to 0} \frac{x - \sin x}{x \tan x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{1 - \cos x}{x \sec^2 x + \tan x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{-(-\sin x)}{(x \cdot 2 \sec x \cdot \sec x \tan x + \sec^2 x) + \sec^2 x}$$

$$= \frac{0}{0 + 1 + 1} = 0$$

48.) $$\lim_{x\to 0} \frac{(e^x - 1)^2}{x \sin x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2(e^x - 1)e^x}{x \cos x + \sin x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{2(e^x - 1)e^x + 2e^x \cdot e^x}{(x \cdot (-\sin x) + \cos x) + \cos x}$$

$$= \frac{0+2}{0+1+1} = 1$$

49.) $\lim\limits_{\theta \to 0} \dfrac{\theta - \sin\theta \cos\theta}{\tan\theta - \theta}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\theta \to 0} \dfrac{1 - (\sin\theta \cdot (-\sin\theta) + \cos\theta \cdot \cos\theta)}{\sec^2\theta - 1}$

$= \lim\limits_{\theta \to 0} \dfrac{1 + \sin^2\theta - \cos^2\theta}{\sec^2\theta - 1}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\theta \to 0} \dfrac{2\sin\theta\cos\theta - 2\cos\theta \cdot (-\sin\theta)}{2\sec\theta \cdot \sec\theta\tan\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{4\sin\theta\cos\theta}{2\sec^2\theta\tan\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{2\sin\theta\cos\theta}{\sec^2\theta\tan\theta}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{\theta \to 0} \dfrac{2\sin\theta(-\sin\theta) + 2\cos\theta\cos\theta}{\sec^2\theta\sec^2\theta + 2\sec^2\theta\tan\theta \cdot \tan\theta}$

$$= \frac{0+2}{1+0} = 2$$

50.) $\lim\limits_{x \to 0} \dfrac{\sin 3x - 3x + x^2}{\sin x \sin 2x}$

$$\overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{3\cos 3x - 3 + 2x}{\sin x \cdot 2\cos 2x + \cos x \sin 2x}$$

$$\overset{"\frac{0}{0}"}{=} \lim_{x\to 0} \frac{-9\sin 3x + 2}{\cos x \cdot 2\cos 2x - 4\sin x \sin 2x - \sin x \sin 2x + 2\cos x \cos 2x}$$

$$= \frac{0+2}{2-0-0+2} = \frac{1}{2}$$

51.) $\lim_{x\to 1^+} x^{\frac{1}{1-x}} = "1^{\frac{1}{0^+}}" = "1^{\infty}"$

$$= \lim_{x\to 1^+} e^{\ln x^{1/(1-x)}} = \lim_{x\to 1^+} e^{\frac{1}{1-x}\ln x}$$

$$= e^{\lim_{x\to 1^+} \frac{\ln x}{1-x}} \overset{"\frac{0}{0}"}{=} e^{\lim_{x\to 1^+} \frac{\frac{1}{x}}{-1}} = e^{-1}$$

52.) $\lim_{x\to 1^-} x^{\frac{1}{1-x}} = "1^{\frac{1}{0^-}}" = "1^{-\infty}" = "\frac{1}{1^{\infty}}"$

$$= \lim_{x\to 1^-} e^{\ln x^{1/(1-x)}} = \lim_{x\to 1^-} e^{\frac{1}{1-x}\ln x}$$

$$= e^{\lim_{x\to 1^-} \frac{\ln x}{1-x}} \overset{"\frac{0}{0}"}{=} e^{\lim_{x\to 1^-} \frac{\frac{1}{x}}{-1}} = e^{-1}$$

53.) $\lim_{x\to\infty} (\ln x)^{1/x} = \text{"}\infty^0\text{"}$

$= \lim_{x\to\infty} e^{\ln (\ln x)^{1/x}} = \lim_{x\to\infty} e^{\frac{1}{x}\ln(\ln x)}$

$= \lim_{x\to\infty} e^{\frac{\ln(\ln x)}{x}} = e^{\lim_{x\to\infty} \frac{\ln(\ln x)}{x}}$

$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1}} = e^0 = 1$

56.) $\lim_{x\to\infty} x^{1/\ln x} = \text{"}\infty^0\text{"}$

$= \lim_{x\to\infty} e^{\ln x^{1/\ln x}} = \lim_{x\to\infty} e^{\frac{1}{\ln x}\cdot \ln x}$

$= \lim_{x\to\infty} e^1 = e$

57.) $\lim_{x\to\infty} (1+2x)^{\frac{1}{2\ln x}} = \text{"}\infty^0\text{"}$

$= \lim_{x\to\infty} e^{\ln (1+2x)^{1/2\ln x}} = \lim_{x\to\infty} e^{\frac{1}{2\ln x}\cdot \ln(1+2x)}$

$= e^{\lim_{x\to\infty} \frac{\ln(1+2x)}{2\ln x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1+2x}\cdot\cancel{2}}{\cancel{2}\cdot 1/x}}$

$= e^{\lim_{x\to\infty} \frac{x}{1+2x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{1}{2}} = e^{1/2}$

58.) $\lim_{x\to 0} (e^x + x)^{1/x} = \text{"}1^{\pm\infty}\text{"}$

$$= \lim_{x\to 0} e^{\ln (e^x+x)^{1/x}} = \lim_{x\to 0} e^{\frac{1}{x}\ln(e^x+x)}$$

$$= e^{\lim_{x\to 0} \frac{\ln(e^x+x)}{x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x\to 0} \frac{\frac{1}{e^x+x}\cdot(e^x+1)}{1}}$$

$$= e^{\frac{2}{1+0}} = e^2$$

59.) $\lim_{x\to 0^+} x^x = \text{"}0^0\text{"} = \lim_{x\to 0^+} e^{\ln x^x}$

$$= \lim_{x\to 0^+} e^{x\ln x} \overset{\text{"}0\cdot\infty\text{"}}{=} e^{\lim_{x\to 0^+} \frac{\ln x}{1/x}}$$

$$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{x}}{-1/x^2}} = e^{\lim_{x\to 0^+}(-x)} = e^0 = 1$$

60.) $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x = \text{"}\infty^0\text{"}$

$$= \lim_{x\to 0^+} e^{\ln\left(1+\frac{1}{x}\right)^x} = \lim_{x\to 0^+} e^{x\ln(1+1/x)}$$

$$\overset{\text{"}0\cdot\infty\text{"}}{=} \lim_{x\to 0^+} e^{\frac{\ln(1+1/x)}{1/x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to 0^+} \frac{\frac{1}{1+1/x}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{-1/x^2}}}$$

$$= e^{\frac{1}{\infty}} = e^0 = 1$$

63.) $\lim\limits_{x\to 0^+} x^2 \ln x \overset{\text{"}0\cdot\infty\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{\ln x}{1/x^2}$

$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{1/x}{2/x^3} = \lim\limits_{x\to 0^+} \frac{1}{2}x^2 = 0$

64.) $\lim\limits_{x\to 0^+} x(\ln x)^2 \overset{\text{"}0\cdot\infty\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{(\ln x)^2}{1/x}$

$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{2\ln x \cdot 1/x}{-1/x^2} = \lim\limits_{x\to 0^+} -2x\ln x$

$= \lim\limits_{x\to 0^+} \dfrac{-2\ln x}{1/x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{-2\cdot 1/x}{-1/x^2}$

$= \lim\limits_{x\to 0^+} 2x = 0$

66.) $\lim\limits_{x\to 0^+} \sin x \cdot \ln x = \text{"}0\cdot\infty\text{"}$

$= \lim\limits_{x\to 0^+} \dfrac{\ln x}{1/\sin x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to 0^+} \dfrac{1/x}{-(\sin x)^{-2}\cdot\cos x}$

$$= \lim_{x\to 0^+} \frac{-(\sin x)^2}{x\cos x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+} \frac{-2\sin x\cdot\cos x}{-x\sin x+(1)\cos x}$$

$$= \frac{-2(\overset{0}{\cancel{\sin 0}})(\overset{1}{\cancel{\cos 0}})}{0+\underset{1}{\cancel{\cos 0}}} = \frac{0}{1} = 0$$

67.) $$\lim_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \sqrt{\frac{9x+1}{x+1}\cdot\frac{1/x}{1/x}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{9+1/x}{1+1/x}} = \sqrt{\frac{9+0}{1+0}} = 3$$

69.) $$\lim_{x\to\frac{\pi}{2}^-} \frac{\sec x}{\tan x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\frac{\pi}{2}^-} \frac{1/\cos x}{\sin x/\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{1}{\sin x} = \frac{1}{\sin\frac{\pi}{2}} = \frac{1}{1} = 1$$

72.) $$\lim_{x\to-\infty} \frac{2^x+4^x}{5^x-2^x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to-\infty} \frac{2^x+4^x}{5^x-2^x}\cdot\frac{1/2^x}{1/2^x}$$

$$= \lim_{x\to-\infty} \frac{1+2^x}{\left(\frac{5}{2}\right)^x-1} = \frac{1+2^{-\infty}}{\left(\frac{5}{2}\right)^{-\infty}-1} = \frac{1+\frac{1}{2^\infty}}{\frac{1}{(5/2)^\infty}-1}$$

$$= \frac{1+0}{0-1} = -1$$

74.) $\lim_{x\to 0^+} \dfrac{x}{e^{-1/x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0^+} \dfrac{e^{1/x}}{1/x}$

$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to 0^+} \dfrac{e^{1/x}\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = e^{1/0^+} = e^{+\infty} = \infty$

79.)

$$f(x) = \begin{cases} \dfrac{9x - 3\sin 3x}{5x^3} & , \text{ if } x \neq 0 \\ c & , \text{ if } x = 0 \end{cases}$$

$$\lim_{x \to 0} \frac{9x - 3\sin 3x}{5x^3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{9 - 3 \cdot \cos 3x \cdot 3}{15x^2}$$

$$= \lim_{x \to 0} \frac{9 - 9\cos 3x}{15x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{-9 \cdot {-\sin 3x} \cdot 3}{30x}$$

$$= \lim_{x \to 0} \frac{27 \sin 3x}{30x} = \lim_{x \to 0} \frac{9 \sin 3x}{10x}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 0} \frac{9 \cdot \cos 3x \cdot 3}{10} = \frac{27(1)}{10} = \frac{27}{10} ;$$

let $c = \frac{27}{10}$; then

i.) $f(0) = \frac{27}{10}$

ii.) $\lim_{x \to 0} f(x) = \frac{27}{10}$

iii.) $\lim_{x \to 0} f(x) = f(0)$

and f is continuous at $x = 0$.

84a.) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = "1^\infty"$

$= \lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x}\right)^x} = \lim_{x\to\infty} e^{x\ln(1+1/x)}$

$= \lim_{x\to\infty} e^{\frac{\ln(1+1/x)}{1/x}}$

$\overset{"\frac{0}{0}"}{=} \lim_{x\to\infty} e^{\frac{\frac{1}{1+1/x}\cdot\cancel{-1/x^2}}{\cancel{-1/x^2}}}$

$= e^{\frac{1}{1+0}} = e$

c.) $\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x = "1^\infty"$

$= \lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x^2}\right)^x} = \lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{x^2}\right)}$

$= \lim_{x\to\infty} e^{\frac{\ln(1+1/x^2)}{1/x}}$

$\overset{"\frac{0}{0}"}{=} \lim_{x\to\infty} e^{\frac{\frac{1}{(1+1/x^2)}\cdot\frac{-2}{x^3}}{-1/x^2}}$

$= \lim_{x\to\infty} e^{\frac{1}{1+1/x^2}\cdot\frac{2}{x}} = e^{\frac{1}{1+0}(0)} = e^0 = 1$

86) b.) $Y = X^{1/x^2}$ for $x > 0$ $\longrightarrow$

$\ln Y = \ln X^{1/x^2} = \frac{1}{x^2} \ln X \xrightarrow{D}$

$\frac{1}{Y} Y' = \frac{x^2 \cdot \frac{1}{x} - 2x \cdot \ln x}{(x^2)^2} \longrightarrow$

$Y' = Y \cdot \frac{x - 2x \ln x}{x^4} = x^{1/x^2} \cdot \frac{x(1 - 2\ln x)}{x^4}$

$= x^{1/x^2} \cdot \frac{1 - 2\ln x}{x^3} = 0 \longrightarrow 1 - 2\ln x = 0$

$\longrightarrow \ln x = \frac{1}{2} \longrightarrow x = e^{1/2}$

$+$ 0 $-$ Y'

$x = 0$ $x = e^{1/2}$

$\left. Y = (e^{1/2})^{1/e} \right\}$ abs. max

87.) b.) $\lim_{x \to \infty} \frac{3x + e^{2x}}{2x + e^{3x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \to \infty} \frac{3 + 2e^{2x}}{2 + 3e^{3x}}$

$\overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \to \infty} \frac{4e^{2x}}{9e^{3x}} = \lim_{x \to \infty} \frac{4}{9} \cdot \frac{1}{e^x}$

$= \frac{4}{9} \cdot \frac{1}{\infty} = \frac{4}{9}(0) = 0$, so

$\boxed{Y = 0}$ is H.A. ;

$$\lim_{x\to-\infty} \frac{3x+e^{2x}}{2x+e^{3x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to-\infty} \frac{3+2e^{2x}}{2+3e^{3x}}$$

$$= \frac{3+2e^{-\infty}}{2+3e^{-\infty}} = \frac{3+2(0)}{2+3(0)} = \frac{3}{2},$$

so $\boxed{Y=\frac{3}{2}}$ is H.A.

88.) $f(x)=\begin{cases} e^{-1/x^2}, & x\neq 0 \\ 0, & x=0 \end{cases}$, then

$$f'(0)=\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)-0}{h} = \lim_{h\to 0} \frac{e^{-1/h^2}}{h}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h\to 0} \frac{1/h}{e^{1/h^2}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{h\to 0} \frac{-1/h^2}{e^{1/h^2}\cdot \frac{-2}{h^3}}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}h}{e^{1/h^2}} = \frac{\frac{1}{2}(0)}{e^{1/0^+}} = \frac{0}{e^{+\infty}} = \frac{0}{\infty} = 0$$