Section 3.2

1.) $f(x) = 4 - x^2 \xrightarrow{D}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(4-(x+h)^2) - (4-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4} - \cancel{x^2} - 2hx - h^2 - \cancel{4} + \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = -2x-(0) = -2x,$$

i.e., $f'(x) = -2x$;

$f'(-3) = 6$, $f'(0) = 0$, $f'(1) = -2$

6.) $r(s) = \sqrt{2s+1}$ so $\xrightarrow{D}$

$$r'(s) = \lim_{h\to 0} \frac{r(s+h) - r(s)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(s+h)+1} - \sqrt{2s+1}}{h} \cdot \frac{\sqrt{2s+2h+1} + \sqrt{2s+1}}{\sqrt{2s+2h+1} + \sqrt{2s+1}}$$

$$= \lim_{h\to 0} \frac{(\cancel{2s} + 2h + \cancel{1}) - (\cancel{2s} + \cancel{1})}{h\cdot(\sqrt{2s+2h+1} + \sqrt{2s+1})}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2s+2h+1} + \sqrt{2s+1})} = \frac{\cancel{2}}{\cancel{2}\sqrt{2s+1}}, \text{ i.e.,}$$

$r'(s) = \dfrac{1}{\sqrt{2s+1}}$; then

$r'(0) = 1$, $r'(1) = \dfrac{1}{\sqrt{3}}$, $r'(\frac{1}{2}) = \dfrac{1}{\sqrt{2}}$.

8.) $r = s^3 - 2s^2 + 3 \xrightarrow{D}$

$$r' = \lim_{h\to 0} \frac{r(s+h) - r(s)}{h}$$

$$= \lim_{h\to 0} \frac{(s+h)^3 - 2(s+h)^2 + \cancel{3} - (s^3 - 2s^2 + \cancel{3})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{s^3} + 3s^2h + 3sh^2 + h^3 - 2(s^2 + 2sh + h^2) - \cancel{s^3} + 2s^2}{h}$$

$$= \lim_{h\to 0} \frac{3s^2h + 3sh^2 + h^3 - \cancel{2s^2} - 4sh - 2h^2 + \cancel{2s^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3s^2 + 3sh + h^2 - 4s - 2h)}{\cancel{h}}$$

$$= 3s^2 + 0 + 0 - 4s - 0 = 3s^2 - 4s$$

13.) $f(x) = x + \frac{9}{x}$ so

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left((x+h) + \frac{9}{x+h}\right) - \left(x + 9/x\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + h + \frac{9}{x+h} - \cancel{x} - \frac{9}{x}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\cancel{h}}{\cancel{h}} + \frac{\frac{9}{x+h} - \frac{9}{x}}{h/1}\right)$$

$$= \lim_{h\to 0} \left(1 + \frac{9x - 9(x+h)}{(x+h)x} \cdot \frac{1}{h}\right)$$

$$= \lim_{h \to 0} \left(1 + \frac{\cancel{9x} - \cancel{9x} - 9h}{(x+h)x \cdot h} \right)$$

$$= \lim_{h \to 0} \left(1 - \frac{9\cancel{h}}{(x+h)x \cdot \cancel{h}} \right) = 1 - \frac{9}{x^2}, \text{ i.e.,}$$

$$f'(x) = 1 - \frac{9}{x^2} \quad \text{so } f'(-3) = 1 - \frac{9}{9} = 0$$

16.) $f(x) = \dfrac{x+3}{1-x} \xrightarrow{D}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{(x+h)+3}{1-(x+h)} - \dfrac{x+3}{1-x}}{h/1}$$

$$= \lim_{h \to 0} \frac{(x+h+3)(1-x) - (1-x-h)(x+3)}{(1-x-h)(1-x)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+h+\cancel{3}-\cancel{x^2}-\cancel{hx}-\cancel{3x} - (\cancel{x}-\cancel{x^2}-\cancel{hx}+\cancel{3}-\cancel{3x}-3h)}{(1-x-h)(1-x) \cdot h}$$

$$= \lim_{h \to 0} \frac{4\cancel{h}}{(1-x-h)(1-x)\cancel{h}} = \frac{4}{(1-x)^2}, \text{ i.e.,}$$

$$f'(x) = \frac{4}{(1-x)^2}, \quad \text{so } f'(-2) = \frac{4}{9}$$

18.) $g(z) = 1 + \sqrt{4-z} \xrightarrow{D}$

$$g'(z) = \lim_{h \to 0} \frac{g(z+h) - g(z)}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{1} + \sqrt{4-(z+h)}) - (\cancel{1} + \sqrt{4-z})}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4-z-h} - \sqrt{4-z}}{h} \cdot \frac{\sqrt{4-z-h} + \sqrt{4-z}}{\sqrt{4-z-h} + \sqrt{4-z}}$$

$$= \lim_{h \to 0} \frac{(\cancel{4} - \cancel{z} - h) - (\cancel{4} - \cancel{z})}{h(\sqrt{4-z-h} + \sqrt{4-z})}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(\sqrt{4-z-h} + \sqrt{4-z})}$$

$$= \frac{-1}{\sqrt{4-z} + \sqrt{4-z}} = \frac{-1}{2\sqrt{4-z}}, \text{ i.e.,}$$

$g'(z) = \dfrac{-1}{2\sqrt{4-z}}$, so SLOPE

$m = g'(3) = \dfrac{-1}{2}$ and $w = 3$, so

tangent line is

$$w - 2 = \frac{-1}{2}(z-3) \quad \text{or}$$

$$w = \frac{-1}{2}z + \frac{7}{2}$$

22.) $w = z + \sqrt{z} \xrightarrow{D}$

$$\frac{dw}{dz} = \lim_{h \to 0} \frac{w(z+h) - w(z)}{h}$$

$$= \lim_{z \to 0} \frac{\cancel{z} + h + \sqrt{z+h} - (\cancel{z} + \sqrt{z})}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{h}{h} + \frac{\sqrt{z+h} - \sqrt{z}}{h} \cdot \frac{\sqrt{z+h} + \sqrt{z}}{\sqrt{z+h} + \sqrt{z}} \right\}$$

$$= \lim_{h \to 0} \left\{ 1 + \frac{\cancel{z} + h - \cancel{z}}{h(\sqrt{z+h} + \sqrt{z})} \right\}$$

$$= \lim_{h \to 0} \left\{ 1 + \frac{\cancel{h}}{\cancel{h}(\sqrt{z+h} + \sqrt{z})} \right\}$$

$$= 1 + \frac{1}{2\sqrt{z}}, \text{ i.e., } \frac{dw}{dz} = 1 + \frac{1}{2\sqrt{z}};$$

$$\left.\frac{dw}{dz}\right|_{z=4} = 1 + \frac{1}{2\sqrt{4}} = 1 + \frac{1}{4} = \frac{5}{4}$$

24.) $f(x) = x^2 - 3x + 4 \xrightarrow{D}$

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

$$= \lim_{z \to x} \frac{z^2 - 3z + \cancel{4} - (x^2 - 3x + \cancel{4})}{z - x}$$

$$= \lim_{z \to x} \frac{z^2 - x^2 - 3z + 3x}{z - x}$$

$$= \lim_{z \to x} \frac{(z-x)(z+x) - 3(z-x)}{z - x}$$

$$= \lim_{z \to x} \frac{\cancel{(z-x)}(z + x - 3)}{\cancel{z-x}}$$

$$= (x) + x - 3 = 2x - 3, \text{ i.e.,}$$

$$f'(x) = 2x - 3$$

25.) $g(x) = \dfrac{x}{x-1} \xrightarrow{D}$

$$g'(x) = \lim_{z \to x} \frac{g(z) - g(x)}{z - x}$$

$$= \lim_{z \to x} \frac{\dfrac{z}{z-1} - \dfrac{x}{x-1}}{\dfrac{z-x}{1}}$$

$$= \lim_{z \to x} \frac{z(x-1) - x(z-1)}{(z-1)(x-1)} \cdot \frac{1}{z-x}$$

$$= \lim_{z \to x} \frac{\cancel{xz} - z - \cancel{xz} + x}{(z-1)(x-1)} \cdot \frac{1}{z-x}$$

$$= \lim_{z \to x} \frac{-\cancel{(z-x)}}{(z-1)(x-1)\cancel{(z-x)}}$$

$$= \frac{-1}{((x)-1)(x-1)} = \frac{-1}{(x-1)^2}, \text{ i.e.,}$$

$$g'(x) = \frac{-1}{(x-1)^2}$$

26.) $g(x) = 1 + \sqrt{x} \xrightarrow{D}$

$$g'(x) = \lim_{z \to x} \frac{g(z) - g(x)}{z - x}$$

$$= \lim_{z \to x} \frac{\cancel{1} + \sqrt{z} - (\cancel{1} + \sqrt{x})}{z - x}$$

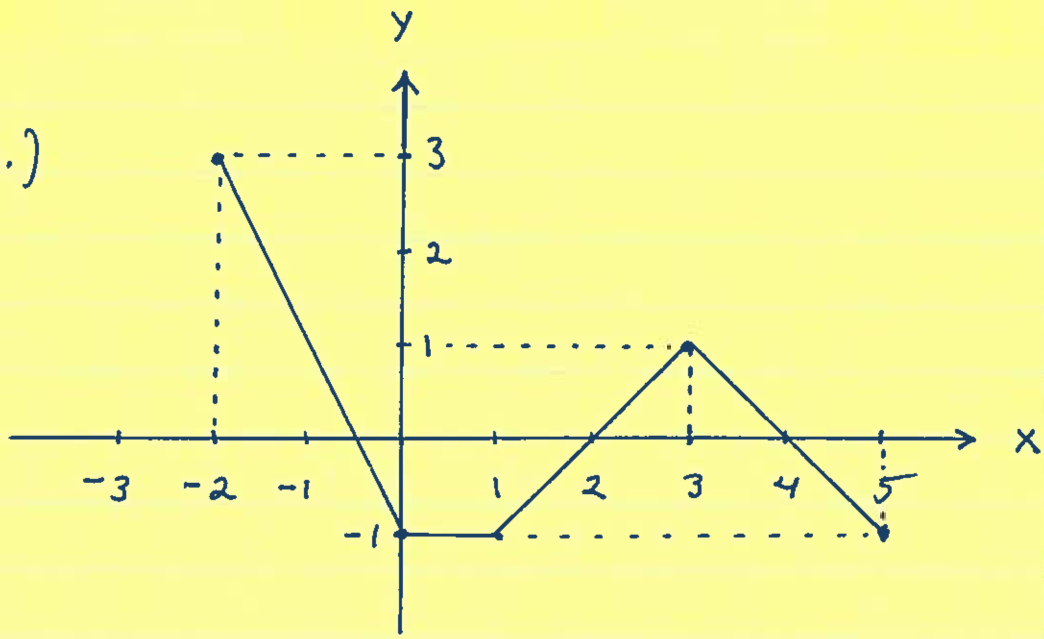
$$= \lim_{z \to x} \frac{\cancel{\sqrt{z} - \sqrt{x}}}{\cancel{(\sqrt{z} - \sqrt{x})}(\sqrt{z} + \sqrt{x})} = \frac{1}{\sqrt{(x)} + \sqrt{x}},$$

i.e., $g'(x) = \frac{1}{2\sqrt{x}}$.

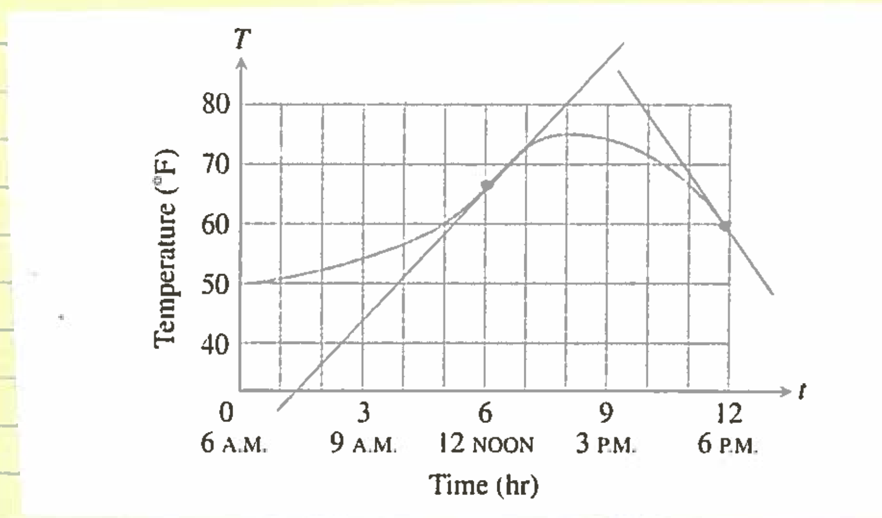
27.) b.) 28.) a.)

29.) d.) 30.) c.)

32.) a.)



35.)



a.) i.) $t=7 \rightarrow T \approx 51°F$

ii.) $t=9 \rightarrow T \approx 54°F$

iii.) $t=2 \rightarrow T \approx 75°F$

iv.) $t=4 \rightarrow T \approx 72°F$

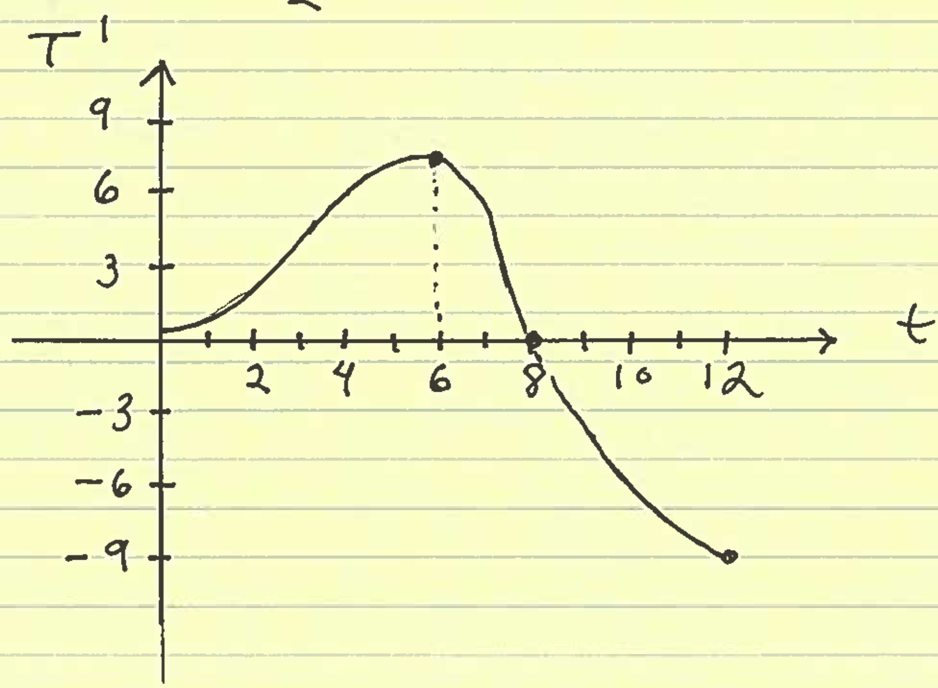
b.) largest ↑ rate at $t \approx$ 12 noon:

RATE $\approx \frac{30}{4} = 7.5$ °F/hr.

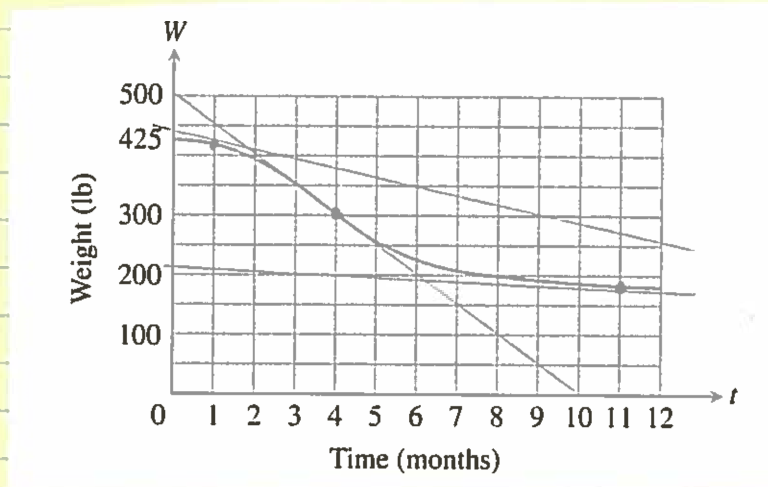
largest ↓ rate at $t \approx$ 6 p.m.:

RATE $\approx \frac{-18}{2} = -9$ °F/hr.

c.



36.)



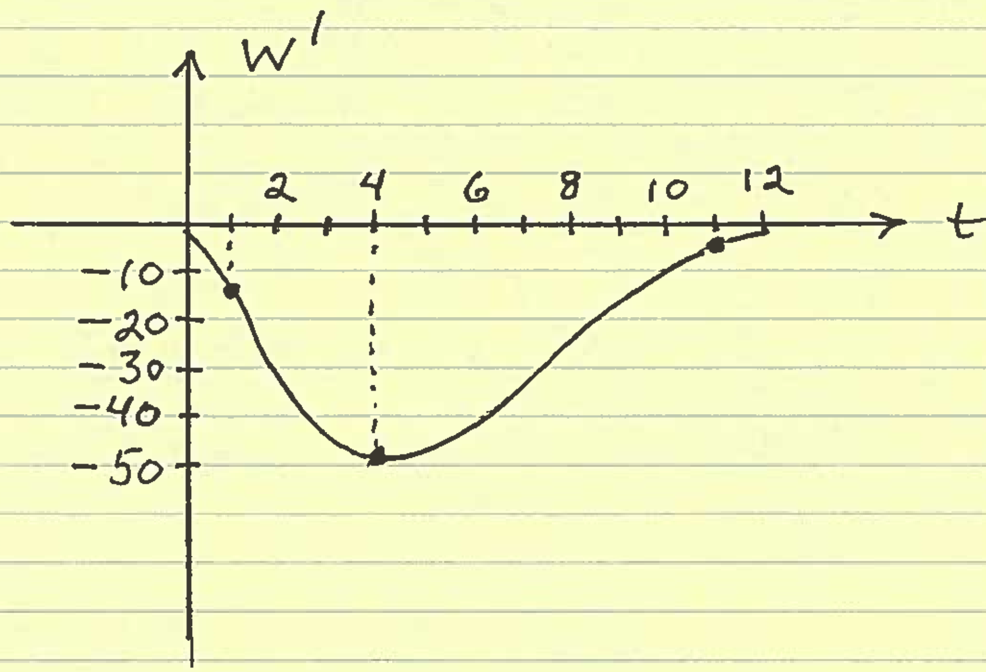
a.) $t=1 \rightarrow \text{RATE} \approx \frac{-190}{12} \approx -15.8$ lbs./mo.

$t=4 \rightarrow \text{RATE} \approx \frac{-500}{10} = -50$ lbs./mo.

$t=11 \rightarrow \text{RATE} \approx \frac{-12}{3} = -4$ lbs./mo.

b.) at $t \approx 4 \rightarrow \text{RATE} \approx -50$ lbs./mo.

c.)



37.) $$f'(0) = \lim_{h\to 0} \frac{f(0+h) - \cancel{f(0)}^{\,0}}{h}$$

$$= \lim_{h\to 0} \frac{f(h)}{h} \quad \text{D.N.E. since}$$

$$\lim_{h\to 0^+} \frac{f(h)}{h} = \lim_{h\to 0^+} \frac{h}{h} = \lim_{h\to 0^+} 1 = 1;$$

$$\lim_{h\to 0^-} \frac{f(h)}{h} = \lim_{h\to 0^-} \frac{h^2}{h} = \lim_{h\to 0^-} h = 0$$

39.) $$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{f(1+h) - 1}{h} \quad \text{D.N.E. since}$$

$$\lim_{h\to 0^+} \frac{f(1+h)-1}{h} = \lim_{h\to 0^+} \frac{(2(1+h)-1)-1}{h}$$

$$= \lim_{h\to 0^+} \frac{\cancel{2}+2h-\cancel{2}}{h} = \lim_{h\to 0^+} 2 = 2;$$

$$\lim_{h\to 0^-} \frac{f(1+h)-1}{h} = \lim_{h\to 0^-} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$$

$$= \lim_{h\to 0^-} \frac{(\cancel{1}+h)-\cancel{1}}{h(\sqrt{1+h}+1)} = \lim_{h\to 0^-} \frac{1}{\sqrt{1+h}+1} = \frac{1}{2}$$

41.) $f(x) = \begin{cases} 2x-1, & x \ge 0 \\ x^2+2x+7, & x<0 \end{cases}$;

$$f'(0) = \lim_{h \to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h)-[2(0)-1]}{h}$$

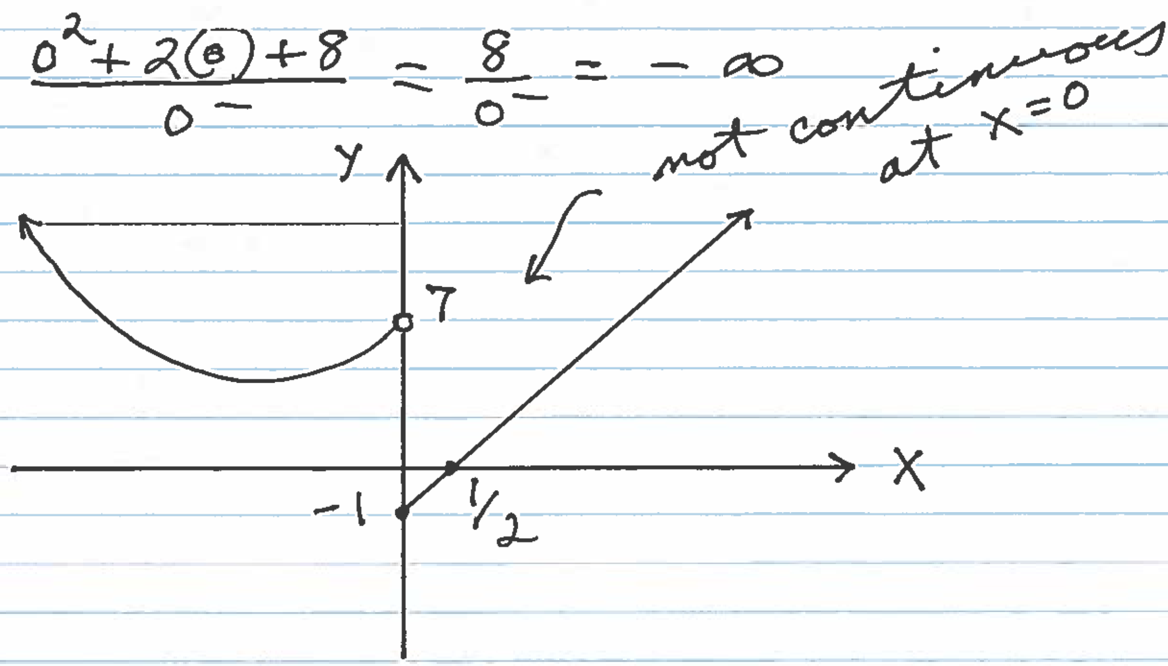
$$= \lim_{h \to 0} \frac{f(h)+1}{h} \quad \text{D.N.E. since}$$

$$\lim_{h \to 0^+} \frac{f(h)+1}{h} = \lim_{h \to 0^+} \frac{2h-\cancel{1}+\cancel{1}}{h}$$

$$= \lim_{h \to 0^+} \frac{2\cancel{h}}{\cancel{h}} = 2 \ ;$$

$$\lim_{h \to 0^-} \frac{f(h)+1}{h} = \lim_{h \to 0^-} \frac{h^2+2h+7+1}{h}$$

$$= \frac{0^2+2(0)+8}{0^-} = \frac{8}{0^-} = -\infty$$



42.) $g(x) = \begin{cases} x^{2/3}, & x \geq 0 \\ x^{1/3}, & x < 0 \end{cases}$;

$$g'(0) = \lim_{h \to 0} \frac{g(0+h) - g(0)}{h}$$

$$= \lim_{h \to 0} \frac{g(h) - (0)^{2/3}}{h}$$

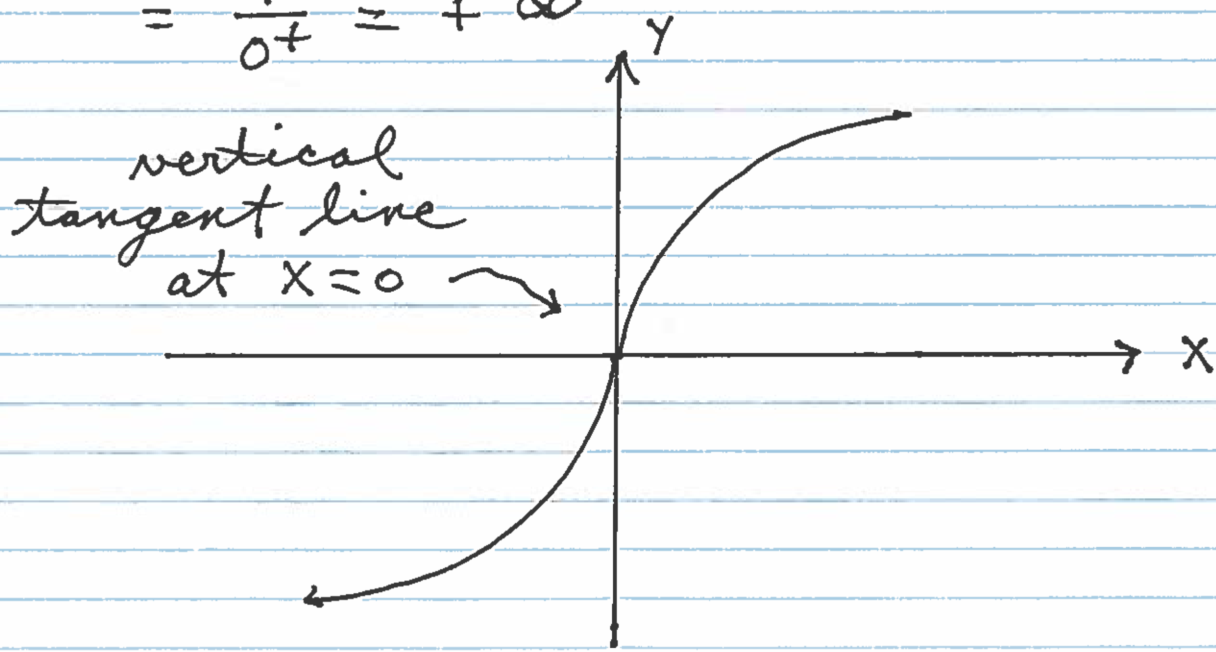
$$= \lim_{h \to 0} \frac{g(h)}{h}$$ D.N.E. since

$$\lim_{h \to 0^+} \frac{g(h)}{h} = \lim_{h \to 0^+} \frac{h^{2/3}}{h} = \lim_{h \to 0^+} \frac{1}{h^{1/3}}$$

$$= \frac{1}{0^+} = +\infty \quad ;$$

$$\lim_{h \to 0^-} \frac{g(h)}{h} = \lim_{h \to 0^-} \frac{h^{1/3}}{h} = \lim_{h \to 0^-} \frac{1}{h^{2/3}}$$

$$= \frac{1}{0^+} = +\infty$$



44.) a.) diff. for $-2 \le x \le 3$

b.) cont., but not diff., for no x-values

c.) neither cont. nor diff. for no x-values

46.) a.) diff for $-2 \le x < -1$, $-1 < x < 0$, $0 < x < 2$, $2 < x \le 3$

b.) cont., but not diff., for $x = -1$

c.) neither cont. nor diff. for $x = 0$, $x = 2$

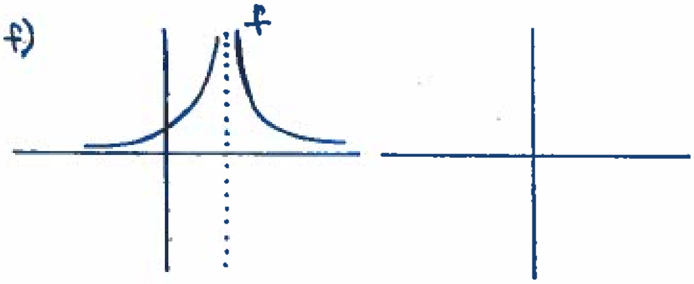
47.) a.) diff. for $-1 \le x < 0$, $0 < x \le 2$

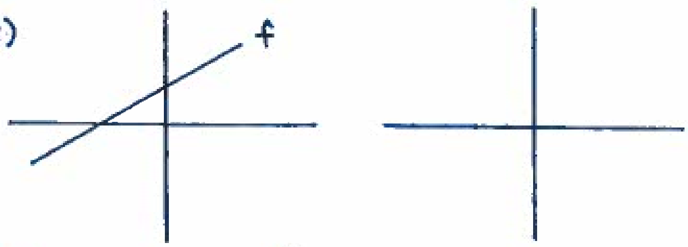
b.) cont., but not diff., for $x = 0$

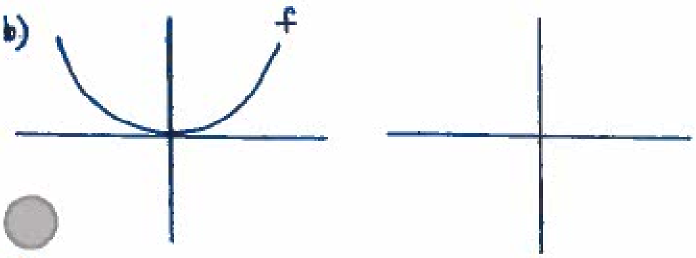
c.) neither cont. nor diff. for no x-values

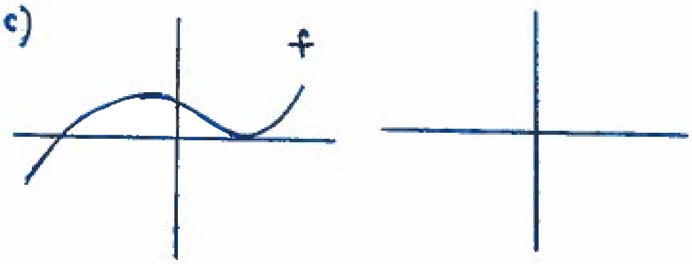
Math 21A
Kouba
Worksheet 4

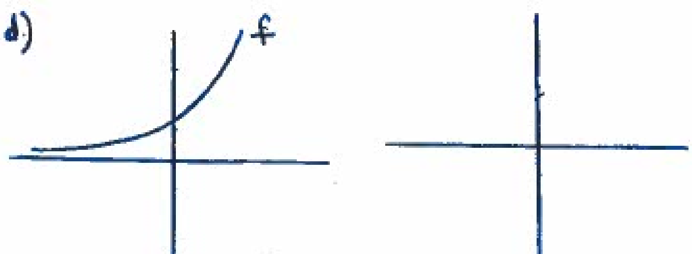
1. Use the given graph of function f to sketch a graph of its derivative, f'.

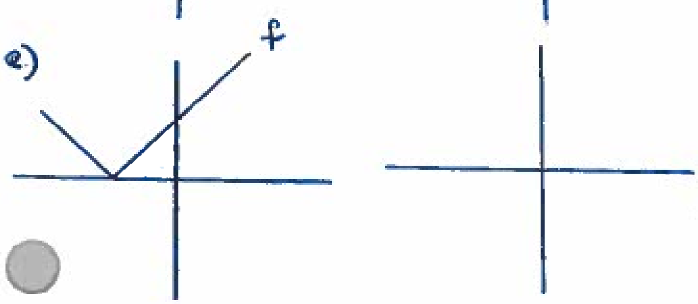
f)


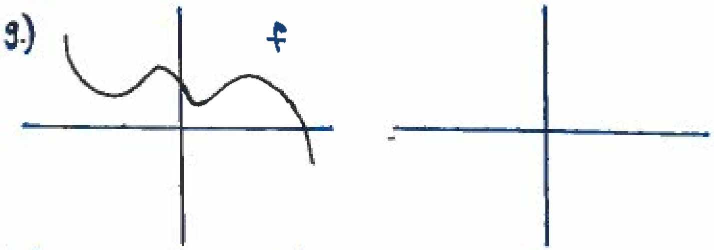
a)


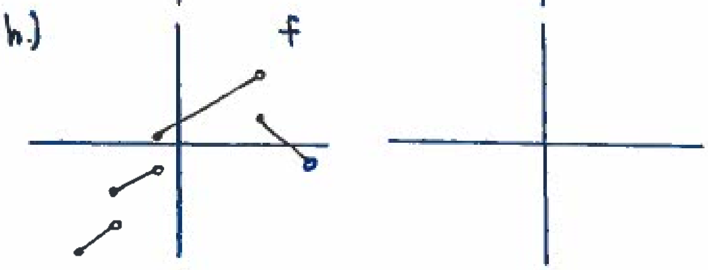
b)


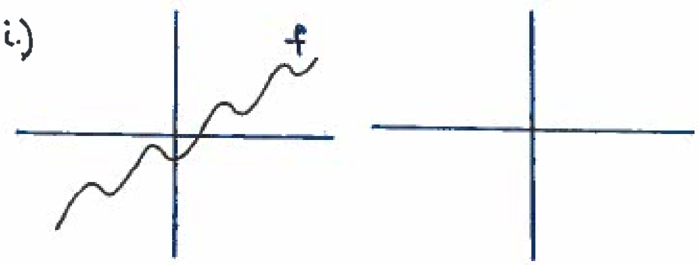
c)


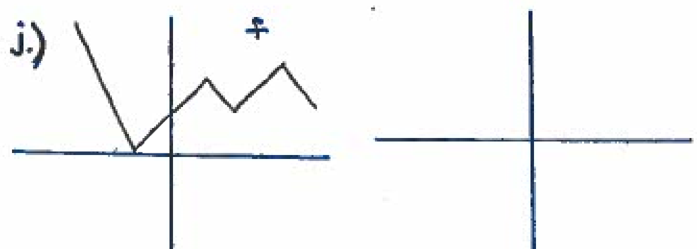
d)


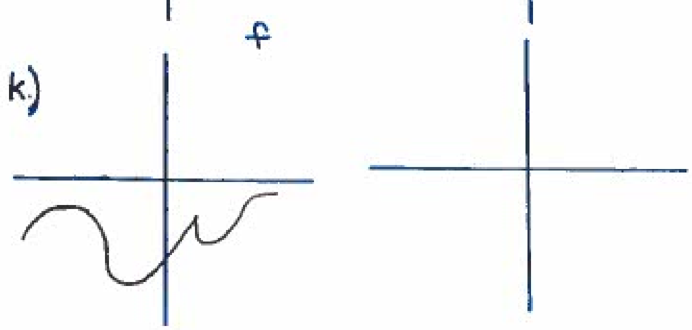
e)


g.)


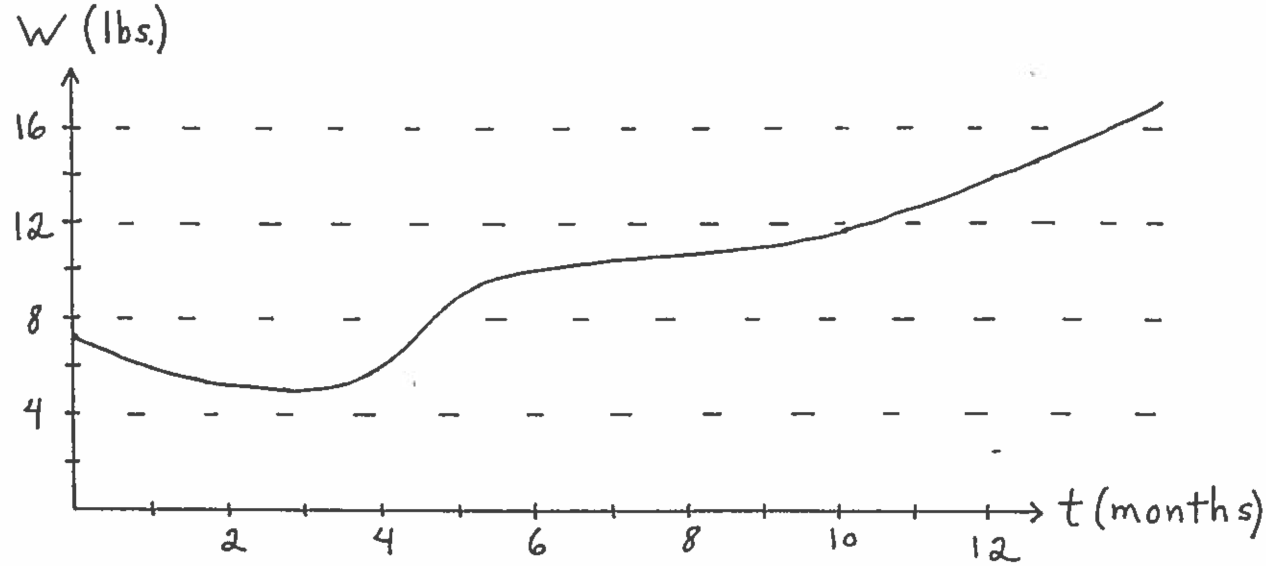
h.)


i.)


j.)


k.)


2. The following chart represents the weight W (lbs.) of a newborn baby as a function of time t (months).



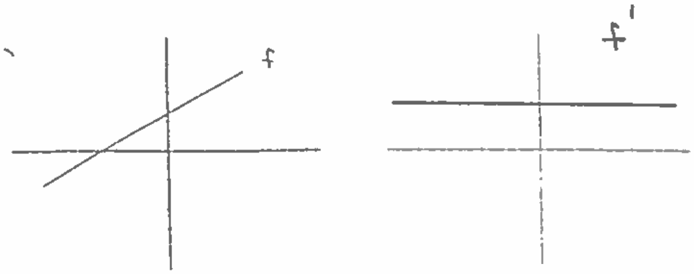
a. What is the baby's weight at birth ? after 3 months ? after 1 year ?

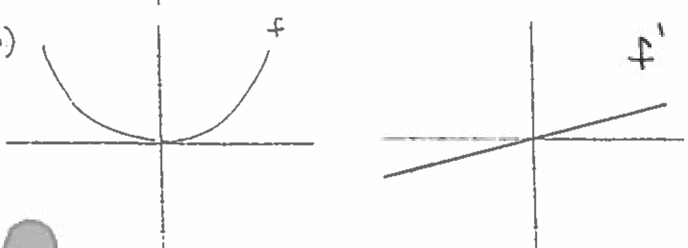
b. What is an estimate of the baby's growth rate (lbs./month) at birth ? after 3 months ? after 1 year ?

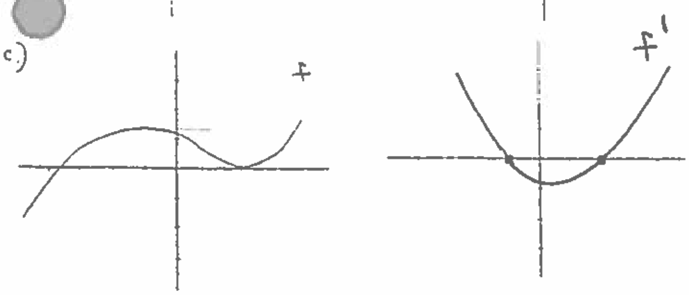
c. When is the baby growing at the fastest rate during its first year of life and what is an estimate for this rate ?

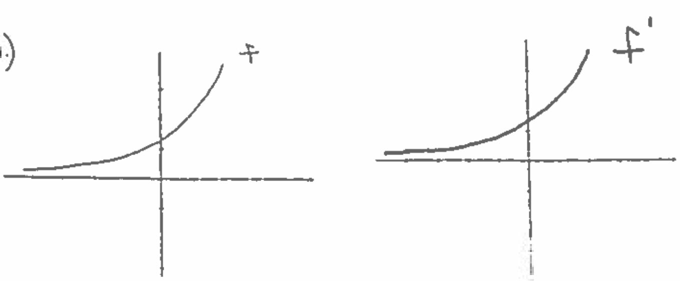
Math 21A
Kouba
Worksheet 4

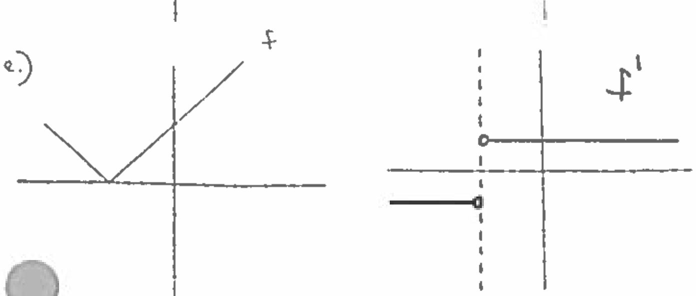
1. Use the given graph of function f to sketch a graph of its derivative, f'.

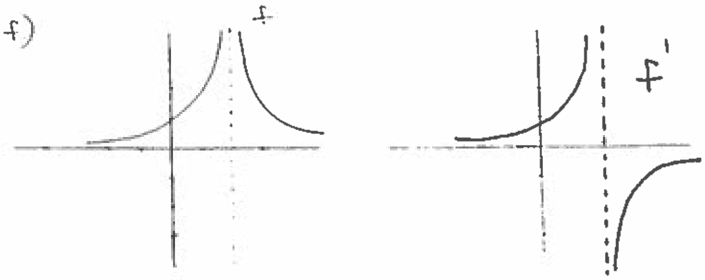
a)


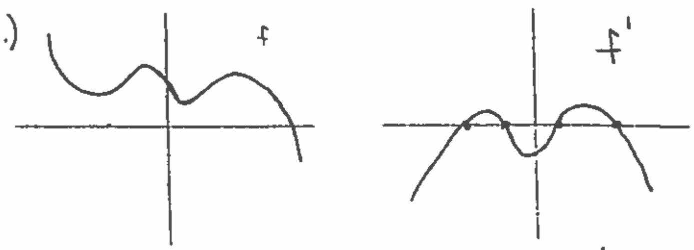
b)


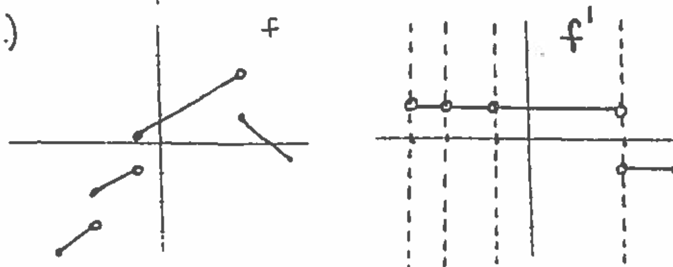
c)


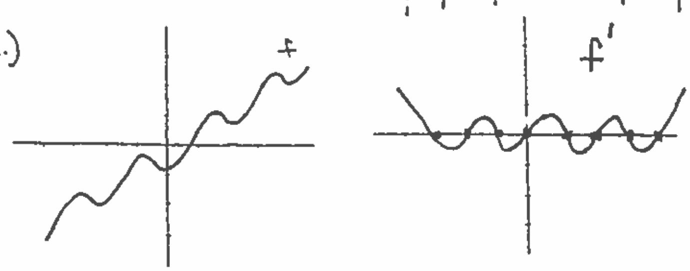
d.)


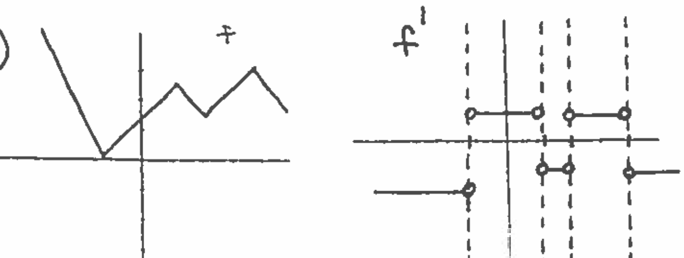
e.)


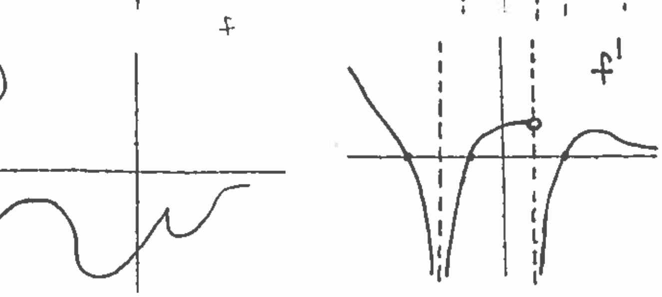
f)


g.)


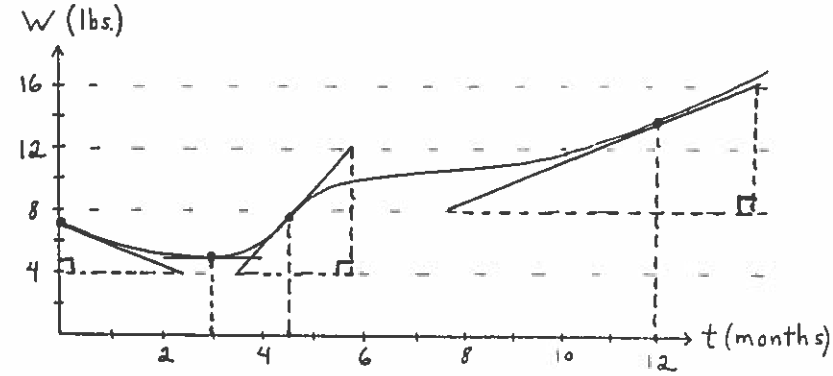
h.)


i.)


j.)


k.)


2.)



a.) $t=0 \rightarrow W = 7$ lbs. , $t = 3$ mo. $\rightarrow W = 5$ lbs. ,
$t = 1$ yr. $\rightarrow W = 14$ lbs.

b.) growth rate : slope of tangent line
$t = 0 \rightarrow$ slope $= \frac{-3}{2.5} = -1.2$ lbs./mo. ,

$t = 3$ mo. $\rightarrow$ slope $= 0$ lbs./mo. ,

$t = 1$ yr. $\rightarrow$ slope $= \frac{8}{6.5} = 1.23$ lbs./mo.

c.) The baby is growing at the fastest rate when $t = 4\frac{1}{2}$ months. The growth rate is

$\frac{8}{2.5} = 3.2$ lbs./mo.