Section 3.3

2.) $y = x^2 + x + 8 \xrightarrow{D} y' = 2x + 1$
$\xrightarrow{D} y'' = 2$

7.) $w = 3z^{-2} - \frac{1}{z} = 3 \cdot z^{-2} - z^{-1} \xrightarrow{D}$
$w' = -6z^{-3} + z^{-2} \xrightarrow{D} w'' = 18z^{-4} - 2z^{-3}$

8.) $s = -2t^{-1} + \frac{4}{t^2} = -2t^{-1} + 4t^{-2} \xrightarrow{D}$
$s' = 2t^{-2} - 8t^{-3} \xrightarrow{D} s'' = -4t^{-3} + 24t^{-4}$

14.) a.) $y = (2x+3)(5x^2 - 4x) \xrightarrow{D}$
$y' = (2x+3)(10x-4) + (2)(5x^2 - 4x)$

16.) a.) $y = (1+x^2)(x^{3/4} - x^{-3}) \xrightarrow{D}$
$y' = (1+x^2)(\frac{3}{4}x^{-1/4} + 3x^{-4}) + (2x)(x^{3/4} - x^{-3})$

17.) $y = \dfrac{2x+5}{3x-2} \xrightarrow{D}$

$$y' = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2}$$

20.) $f(t) = \dfrac{t^2-1}{t^2+t-2} \xrightarrow{D}$

$$f'(t) = \frac{(t^2+t-2)(2t) - (t^2-1)(2t+1)}{(t^2+t-2)^2}$$

23.) $f(s) = \dfrac{\sqrt{s}-1}{\sqrt{s}+1} \xrightarrow{D}$

$$f'(s) = \frac{(\sqrt{s}+1)\left(\frac{1}{2}s^{-1/2}\right) - (\sqrt{s}-1)\cdot\left(\frac{1}{2}s^{-1/2}\right)}{(\sqrt{s}+1)^2}$$

28.) $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)} \xrightarrow{D}$

$$y' = \frac{(x-1)(x-2)\cdot D\{(x+1)(x+2)\} - (x+1)(x+2)\cdot D\{(x-1)(x-2)\}}{((x-1)(x-2))^2}$$

$$= \frac{(x-1)(x-2)\cdot\{(x+1)(1) + (1)(x+2)\} - (x+1)(x+2)\cdot\{(x-1)(1) + (1)(x-2)\}}{(x-1)^2(x-2)^2}$$

31.) $y = x^3 e^x \xrightarrow{D}$

$y' = x^3 \cdot e^x + 3x^2 \cdot e^x$

34.) $y = x^{-3/5} + \pi^{3/2} \xrightarrow{D}$

$y' = -\frac{3}{5} x^{-8/5} + 0$

39.) $r = \dfrac{e^s}{s} \xrightarrow{D} r' = \dfrac{s \cdot e^s - e^s \cdot (1)}{s^2}$

42.) $Y = \frac{1}{120} x^5 \xrightarrow{D} Y' = \frac{1}{120} \cdot 5x^4 = \frac{1}{24} x^4$

$\xrightarrow{D} Y'' = \frac{1}{24} \cdot 4x^3 = \frac{1}{6} x^3 \xrightarrow{D}$

$Y''' = \frac{1}{6} \cdot 3x^2 = \frac{1}{2} x^2 \xrightarrow{D}$

$Y^{(4)} = \frac{1}{2} \cdot 2x = x \xrightarrow{D} Y^{(5)} = 1 \xrightarrow{D}$

$Y^{(6)} = Y^{(7)} = Y^{(8)} = \cdots = 0$

43.) $y = (x-1)(x+2)(x+3) \xrightarrow{D}$

$$y' = (1)(x+2)(x+3) + (x-1)(1)(x+3) + (x-1)(x+2)(1) \xrightarrow{D}$$

$$y'' = (x+2)(1) + (1)(x+3) + (x-1)(1) + (1)(x+3)$$
$$+ (x-1)(1) + (1)(x+2)$$
$$= 6x + 8 \xrightarrow{D}$$

$$y'' = 6 \xrightarrow{D} y''' = 0 \xrightarrow{D} y^{(4)} = y^{(5)} = y^{(6)} = \cdots = 0$$

46.) $s = \dfrac{t^2 + 5t - 1}{t^2} = 1 + 5t^{-1} - t^{-2} \xrightarrow{D}$

$$s' = 0 - 5t^{-2} + 2t^{-3} \xrightarrow{D}$$

$$s'' = 10t^{-3} - 6t^{-4}$$

51.) $w = 3z^2 e^z \xrightarrow{D}$

$$w' = 3z^2 \cdot e^z + 6z \cdot e^z \xrightarrow{D}$$

$$w'' = (3z^2 \cdot e^z + 6z \cdot e^z)$$
$$+ (6z \cdot e^z + 6e^z)$$

52.) $w = e^z (z-1)(z^2+1) \xrightarrow{D}$ (triple product rule)

$$w' = e^z \cdot (z-1)(z^2+1) + e^z (1)(z^2+1) + e^z (z-1)(2z)$$

$$= e^z [z^3 - \cancel{z^2} + z - \cancel{1} + \cancel{z^2} + \cancel{1} + 2z^2 - 2z]$$

$$= e^z [z^3 + 2z^2 - z] \xrightarrow{D}$$

$$w'' = e^z \cdot (3z^2 + 4z - 1) + e^z (z^3 + 2z^2 - z)$$

$$= e^{z}\left(z^3+5z^2+3z-1\right)$$

54.) a.) $\frac{d}{dx}(uv) = u\cdot v' + u'\cdot v$ (let $x=1$)

$$= u(1)\cdot v'(1) + u'(1)\cdot v(1)$$

$$= (2)(-1) + (0)(5) = -2$$

b.) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot u' - u\cdot v'}{v^2}$ (let $x=1$)

$$= \frac{v(1)\cdot u'(1) - u(1)\cdot v'(1)}{(v(1))^2}$$

$$= \frac{(5)\cdot(0) - (2)\cdot(-1)}{5^2} = \frac{2}{25}$$

c.) $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u\cdot v' - v\cdot u'}{u^2}$ (let $x=1$)

$$= \frac{u(1)\cdot v'(1) - v(1)\cdot u'(1)}{(u(1))^2}$$

$$= \frac{(2)(-1) - (5)(0)}{(2)^2} = \frac{-2}{4} = \frac{-1}{2}$$

55.) a.) $y = x^3 - 4x + 1$ at $(2,1)$ so $y' = 3x^2 - 4$ and slope of tangent line at $(2,1)$ is

$Y' = 3(2)^2 - 4 = 8$; then slope of $\perp$ line is

$m = -1/8$ and equation of $\perp$ line at $(2,1)$ is

$Y - 1 = \frac{-1}{8}(X-2) \rightarrow Y = \frac{-1}{8}X + \frac{5}{4}$

b.) Curve $y = x^3 - 4x + 1$ so SLOPES of curve are

$$m = Y' = 3x^2 - 4 = \underbrace{(3x^2)}_{\geq 0} + -4,$$

so smallest SLOPE is when $x = 0$, $y = 1$, and SLOPE $= 3(0)^2 - 4 = -4$

c.) If SLOPE $y' = 3x^2 - 4 = 8 \rightarrow$
$3x^2 = 12 \rightarrow x^2 = 4 \rightarrow x = \pm 2$;

if $x = 2$, then $y = 8 - 8 + 1 = 1$ and tangent line is

$$Y - 1 = 8(X-2) \text{ or } Y = 8X - 15;$$

if $x = -2$, then $y = -8 + 8 + 1 = 1$ and tangent line is

$$Y - 1 = 8(X - (-2)) \text{ or } Y = 8X + 17$$

57.) $Y = \dfrac{4X}{X^2+1} \xrightarrow{D} Y' = \dfrac{(X^2+1)(4) - 4X(2X)}{(X^2+1)^2}$

$= \dfrac{4X^2+4-8X^2}{(X^2+1)^2} = \dfrac{4-4X^2}{(X^2+1)^2}$

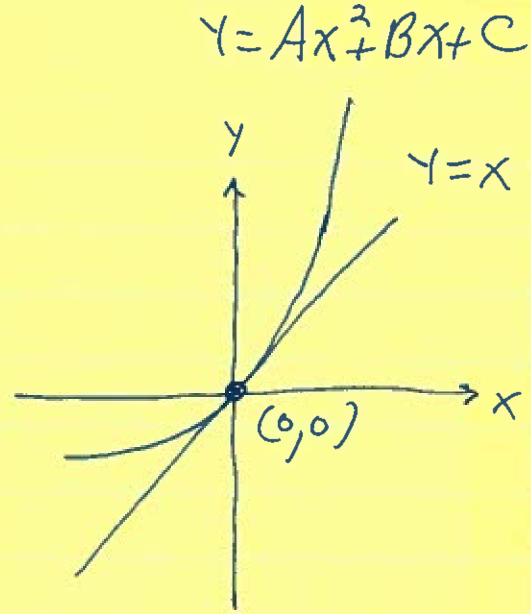
a.) For $(0,0)$ slope of tangent line is $m = Y' = \dfrac{4}{(1)^2} = 4$, so equation of line is

$Y - 0 = 4(X-0) \longrightarrow Y = 4X$.

b.) For $(1,2)$ slope of tangent line is $m = Y' = \dfrac{0}{(1)^2} = 0$, so equation of line is

$Y - 2 = 0(X-1) \longrightarrow Y = 2$.

59.) $Y = Ax^2 + BX + C$ passes through point $\boxed{(1,2)}$; is



~~tangent to~~ $Y = X$ at $(0,0)$ so also passes through $\boxed{(0,0)}$; thus

$Y' = 2AX + B$ and $Y' = 1$ have the same value when $X = 0$, i.e.,

$2A(0) + B = 1 \longrightarrow \boxed{B = 1}$;

thus $Y = AX^2 + X + C$

$(X=1, Y=2)$ $\quad 2 = A + 1 + C \longrightarrow$

$\boxed{A + C = 1}$;

$(X=0, Y=0)$ $\quad 0 = 0 + 0 + C \longrightarrow \boxed{C = 0}$

so $\boxed{A = 1}$ and $\boxed{Y = X^2 + X}$.

61.) $f(X) = 3X^2 - 4X$ and line $Y = 8X + 5$ has slope $m = 8$; then

$f'(X) = 6X - 4 = 8 \rightarrow 6X = 12 \rightarrow X = 2$

and $Y = f(2) = 3(2)^2 - 4(2) = 4$

63.) $f(X) = \dfrac{X}{X-2}$ and line $Y = 2X + 3$

has slope 2 so $\perp$ is $m = -1/2$; then

$$f'(x) = \frac{(x-2)\cdot(1) - x\cdot(1)}{(x-2)^2} = \frac{-2}{(x-2)^2} = \frac{-1}{2}$$

$\rightarrow 4 = (x-2)^2 \rightarrow \sqrt{4} = \sqrt{(x-2)^2}$

$\rightarrow 2 = |x-2| \rightarrow x-2 = 2$ or $x-2 = -2$

$\rightarrow x = 4$ and $y = f(4) = \frac{4}{4-2} = 2$;

or $x = 0$ and $y = f(0) = \frac{0}{0-2} = 0$.

64.) $f(x) = x^2 \xrightarrow{D} f'(x) = 2x$; then

SLOPE of tangent line at (x, y) is

i.) $m = 2x$

and ii.) $m = \frac{8-y}{3-x} = \frac{8-x^2}{3-x}$; then

$\frac{8-x^2}{3-x} = 2x \rightarrow 8 - x^2 = 6x - 2x^2$

$\rightarrow x^2 - 6x + 8 = 0$

$\rightarrow (x-2)(x-4) = 0$

$\rightarrow x = 2$ and $y = f(2) = 4$

or $x = 4$ and $y = f(4) = 16$

66.) a.) $Y = x^3 - 6x^2 + 5x \xrightarrow{D}$

$Y' = 3x^2 - 12x + 5$; at $(0,0)$

slope of tangent line is

$m = Y' = 0 - 0 + 5 = 5$ so

equation of line is

$Y - 0 = 5(x - 0) \rightarrow Y = 5x$

67.) Let $f(x) = x^{50}$, then $f'(x) = 50x^{49}$

and $f'(1) = \lim_{x \to 1} \dfrac{f(x) - f(1)}{x - 1}$

$= \lim_{x \to 1} \dfrac{x^{50} - 1^{50}}{x - 1}$

$= \lim_{x \to 1} \dfrac{x^{50} - 1}{x - 1} = 50(1)^{49} = 50$

68.) Let $f(x) = x^{2/9}$, then $f'(x) = \frac{2}{9} x^{-7/9}$

and $f'(-1) = \lim_{x \to -1} \dfrac{f(x) - f(-1)}{x - (-1)}$

$= \lim_{x \to -1} \dfrac{x^{2/9} - 1}{x + 1} = \frac{2}{9}(-1)^{-7/9} = \frac{-2}{9}$

69.) $g(x) = \begin{cases} ax & , \text{ if } x < 0 \\ x^2 - 3x, & \text{if } x \geq 0 \end{cases}$;

$y = ax$ is diff. for $x < 0$;
$y = x^2 - 3x$ is diff. for $x > 0$;

$$g'(0) = \lim_{h \to 0^+} \frac{g(0+h) - g(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{h^2 - 3h - 0}{h}$$

$$= \lim_{h \to 0^+} \frac{\cancel{h}(h-3)}{\cancel{h}} = 0 - 3 = \boxed{-3};$$

$$g'(0) = \lim_{h \to 0^-} \frac{g(0+h) - g(0)}{h}$$

$$= \lim_{h \to 0^-} \frac{ah - 0}{h}$$

$$= \lim_{h \to 0^-} a = \boxed{a};$$

if $g'(0)$ exists then $\boxed{a = -3}$.

70.) $f(x) = \begin{cases} ax + b, & \text{if } x > -1 \\ bx^2 - 3, & \text{if } x \leq -1 \end{cases}$;

$y = ax + b$ is diff. for $x > -1$;
$y = bx^2 - 3$ is diff. for $x < -1$;

if $f'(-1)$ exists then f is continuous at $x=-1$, so that

$$\lim_{x\to -1^+} f(x) = f(-1) \text{ ; then}$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (ax+b)$$

$$= a(-1)+b = \boxed{b-a} ,$$

and $f(-1) = b(-1)^2 - 3 = \boxed{b-3}$; then

$$\cancel{b} - a = \cancel{b} - 3 \rightarrow \boxed{\boxed{a=3}} \text{ ; also}$$

$$f'(-1) = \lim_{h\to 0^+} \frac{f(-1+h)-f(-1)}{h}$$

$$= \lim_{h\to 0^+} \frac{a(-1+h)+b-(b-3)}{h}$$

$$= \lim_{h\to 0^+} \frac{-a+ah+\cancel{b}-\cancel{b}+3}{h}$$

(and $a=3$)

$$= \lim_{h\to 0^+} \frac{-\cancel{3}+3h+\cancel{3}}{h}$$

$$= \lim_{h\to 0^+} \frac{3\cancel{h}}{\cancel{h}} = \boxed{3} \text{ ;}$$

$$f'(-1) = \lim_{h\to 0^-} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h\to 0^-} \frac{b(-1+h)^2 - 3 - (b-3)}{h}$$

$$= \lim_{h\to 0^-} \frac{b(1-2h+h^2) - \cancel{3} - b + \cancel{3}}{h}$$

$$= \lim_{h\to 0^-} \frac{\cancel{b} - 2bh + bh^2 - \cancel{b}}{h}$$

$$= \lim_{h\to 0^-} \frac{\cancel{h}(-2b + bh)}{\cancel{h}}$$

$$= -2b + b(0) = \boxed{-2b} \; ;$$

if $f'(-1)$ exists, then $-2b = 3$

$$\rightarrow \boxed{b = \frac{-3}{2}}$$

75.) a.) Let $y = u \cdot v \cdot w = (u \cdot v) \cdot w \xrightarrow{D}$

$$y' = (u \cdot v) \cdot w' + D(u \cdot v) \cdot w$$

$$= u \cdot v \cdot w' + (uv' + u'v)w$$

$$= u \cdot v \cdot w' + u \cdot v' \cdot w + u' \cdot v \cdot w$$

77.) $P = \dfrac{nRT}{V-nb} - \dfrac{an^2}{V^2} \xrightarrow{D}$

$$\frac{dP}{dV} = D\left\{\frac{nRT}{V-nb}\right\} - D\left\{\frac{an^2}{V^2}\right\}$$

$$= \frac{(V-nb)(0) - nRT\cdot(1)}{(V-nb)^2} - \frac{V^2\cdot(0) - an^2\cdot 2V}{(V^2)^2}$$

$$= \frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}.$$

78.) $A(q) = km\cdot q^{-1} + cm + \frac{h}{2}\cdot q \xrightarrow{D}$

$$\frac{dA}{dq} = -km\cdot q^{-2} + 0 + \frac{h}{2} \xrightarrow{D}$$

$$\frac{d^2A}{dq^2} = 2km\cdot q^{-3} + 0$$