Math 21A
Komba
an Example Using the Intermediate Value Theorem

Ex: Determine if \( x^3 + x^2 = \sqrt{x+4} \) is solvable.

Begin by sketching \( y = x^3 + x^2 = x^2(x+1) \) and \( y = \sqrt{x+4} \).

It appears that there is a solution. Now we must prove its existence. Since \( x^3 + x^2 = \sqrt{x+4} \) then

\[
x^3 + x^2 - \sqrt{x+4} = 0
\]

so let \( f(x) = x^3 + x^2 - \sqrt{x+4} \) and let \( m = 0 \).

By trial and error \( f(0) = -2 \) and \( f(5) = 147 \), and \( m = 0 \) is between \( f(0) \) and \( f(5) \).

Thus, since \( f \) is a continuous function (it is sum and difference of continuous functions) on a closed interval \([0, 5]\), it follows from the IMVT that there is at least one number \( c \) in \([0, 5]\) so that \( f(c) = m \), i.e.,

\[
c^3 + c^2 - \sqrt{c+4} = 0, \text{ i.e., } c^3 + c^2 = \sqrt{c+4}
\]

Thus, we have proven that the original...