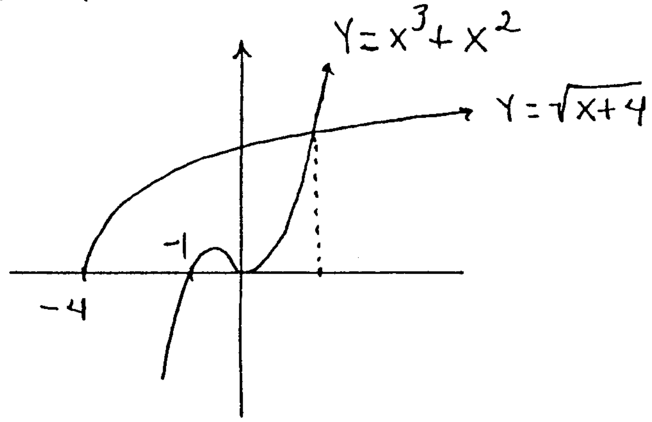
Math 21A
Kouba
An Example Using the Intermediate Value Theorem

IMVT

Ex: Determine if $x^3 + x^2 = \sqrt{x+4}$ is solvable.

Begin by sketching $y = x^3 + x^2 = x^2(x+1)$ and $y = \sqrt{x+4}$:



It *appears* that there is a solution. Now we must *prove* its existance. Since $x^3 + x^2 = \sqrt{x+4}$ then $x^3 + x^2 - \sqrt{x+4} = 0$ so let $f(x) = x^3 + x^2 - \sqrt{x+4}$ and let $m = 0$. By trial and error $f(0) = -2$ and $f(5) = 147$, and $m=0$ is between $f(0)$ and $f(5)$.

Thus, since f is a continuous function (It is sum and difference of continuous functions.) on a closed interval $[0,5]$, it follows from the IMVT that there is at least one number c in $[0,5]$ so that

$$f(c) = m \quad , \text{ i.e.,}$$

$$c^3 + c^2 - \sqrt{c+4} = 0 \quad , \text{ i.e.,}$$

$$c^3 + c^2 = \sqrt{c+4} \; .$$

Thus we have proven that the original