Math 21A
Kouba
Derivatives of Inverse Trig Functions

Trig Function	Domain Restriction	Inverse Function	Derivative of Inverse
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \arctan x$	$y' = \frac{1}{1+x^2}$
$y = \sin x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$y = \arcsin x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \sec x$	$0 \le x \le \pi, x \ne \frac{\pi}{2}$	$y = \operatorname{arcsec} x$	$y' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$y = \cot x$	$0 < x < \pi$	$y = \operatorname{arccot} x$	$y' = \frac{-1}{1+x^2}$
$y = \cos x$	$0 \le x \le \pi$	$y = \arccos x$	$y' = \frac{-1}{\sqrt{1-x^2}}$
$y = \csc x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}, x \ne 0$	$y = \operatorname{arccsc} x$	$y' = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$

Why is $D \arctan x = \dfrac{1}{1+x^2}$?

PROOF : $y = \arctan x \implies x = \tan y$ (Definition of inverse tangent)

$\implies 1 = \sec^2 y \cdot y'$ (Implicit differentiation)

$\implies y' = \dfrac{1}{\sec^2 y}$ (Solve for y.)

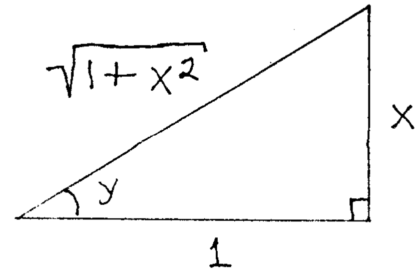
$\implies y' = \dfrac{1}{1/\cos^2 y}$ (Definition of secant)

$\implies y' = \cos^2 y$

$\implies y' = (\cos y)^2$

$\implies y' = \left(\dfrac{1}{\sqrt{1+x^2}}\right)^2$ (Definition of cosine and Pythagorean Theorem from right triangle)

$\implies y' = \dfrac{1}{1+x^2}$



$\tan y = x = \dfrac{x}{1}$