

Math 21A

Kouba

The Plausibility of L'Hopital's Rule, The $\frac{0}{0}$ Case

L'Hopital's Rule ($\frac{0}{0}$ Case) : If $\lim_{x \rightarrow a} f(x) = 0$, $\lim_{x \rightarrow a} g(x) = 0$, and $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = L$

(a finite number or $\pm\infty$), then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = L$.

Assume that f, g, f' , and g' are continuous for all x -values in an interval containing a , so that

$$\lim_{x \rightarrow a} f(x) = f(a) = 0,$$

$$\lim_{x \rightarrow a} g(x) = g(a) = 0,$$

$$\lim_{x \rightarrow a} f'(x) = f'(a)$$

$$\text{and } \lim_{x \rightarrow a} g'(x) = g'(a).$$

Then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)}$$

$$= \lim_{x \rightarrow a} \frac{\frac{f(x) - f(a)}{x - a}}{\frac{g(x) - g(a)}{x - a}}$$

$$= \frac{f'(a)}{g'(a)}$$

$$= \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = L.$$