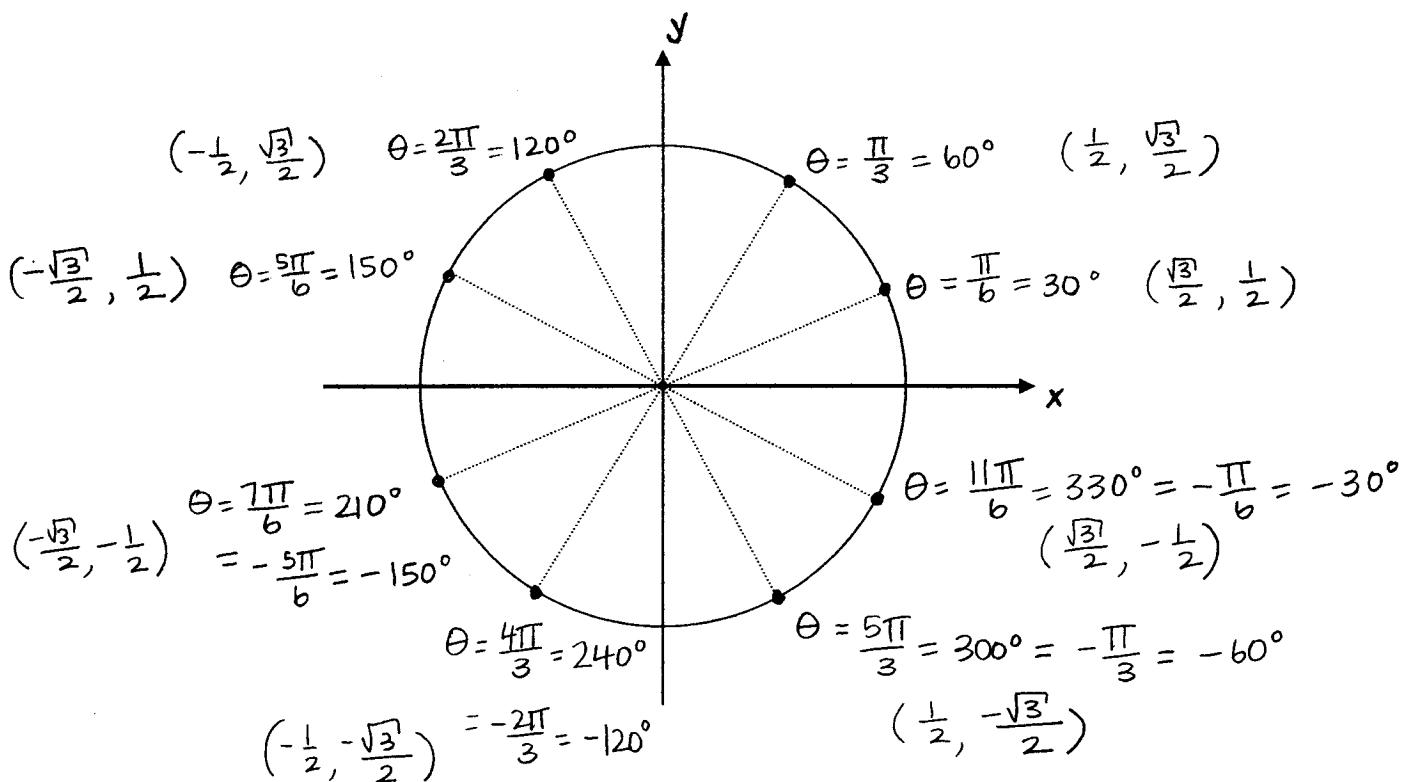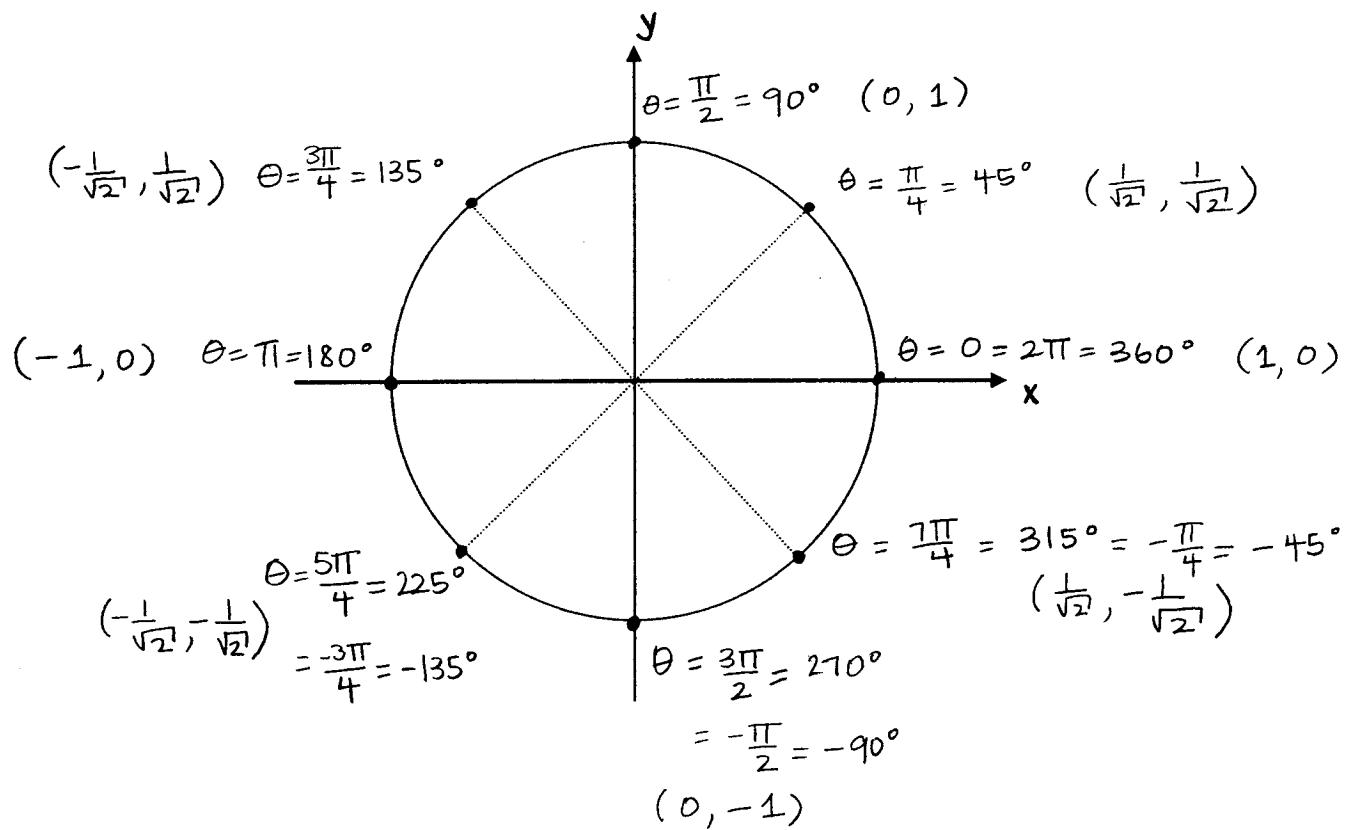Math 21A
Kouba
Exact Trig Values (Please memorize these at your earliest convenience.)

degrees	radians	$\cos \theta$	$\sin \theta$	$\tan \theta$
0 ⁰	0	1	0	0
30 ⁰	$\pi/6$	$\sqrt{3}/2$	$1/2$	$1/\sqrt{3}$
45 ⁰	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60 ⁰	$\pi/3$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}$
90 ⁰	$\pi/2$	0	1	undefined

COSINE AND SINE VALUES ON THE UNIT CIRCLE

$\theta = \frac{\pi}{2} = 90°$ $(0, 1)$

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $\theta = \frac{3\pi}{4} = 135°$

$\theta = \frac{\pi}{4} = 45°$ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$(-1, 0)$ $\theta = \pi = 180°$

$\theta = 0 = 2\pi = 360°$ $(1, 0)$

$\theta = \frac{7\pi}{4} = 315° = -\frac{\pi}{4} = -45°$

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\theta = \frac{5\pi}{4} = 225°$

$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$= \frac{-3\pi}{4} = -135°$

$\theta = \frac{3\pi}{2} = 270°$

$= -\frac{\pi}{2} = -90°$

$(0, -1)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\theta = \frac{2\pi}{3} = 120°$

$\theta = \frac{\pi}{3} = 60°$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\theta = \frac{5\pi}{6} = 150°$

$\theta = \frac{\pi}{6} = 30°$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\theta = \frac{7\pi}{6} = 210°$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $= -\frac{5\pi}{6} = -150°$

$\theta = \frac{11\pi}{6} = 330° = -\frac{\pi}{6} = -30°$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\theta = \frac{4\pi}{3} = 240°$

$\theta = \frac{5\pi}{3} = 300° = -\frac{\pi}{3} = -60°$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $= -\frac{2\pi}{3} = -120°$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Formulas for Use in Trigonometry

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$

$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$

$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$