Assume that an object moves along a graph in the $xy$-plane in such a way that its location $(x,y)$ at time $t$ is given parametrically by

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}.$$ 

The object’s speed at time $t$ is given by

$$\frac{ds}{dt} = \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2}.$$ 

The distance (arc length) the object travels from time $t = t_1$ to time $t = t_2$ is given by

$$\text{ARC} = \int_{t_1}^{t_2} \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \, dt.$$ 

Here are two alternate formulas for arc length:

I.) If a graph is given in rectangular form by $y = f(x)$ from $x = a$ to $x = b$ then

$$\text{ARC} = \int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx.$$ 

II.) If a graph is given in polar form by $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ then

$$\text{ARC} = \int_{\alpha}^{\beta} \sqrt{(f(\theta))^2 + (f'(\theta))^2} \, d\theta.$$