Math 21B (Kouba)
Parametric Equations
Speed, Arc Length

Assume that an object moves along a graph in the xy-plane in such a way that its *LOCATION* (x, y) at time t is given parametrically by

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases} .$$

The object's *SPEED* at time t is given by

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} .$$

The distance (*ARC LENGTH*) the object travels from time $t = t_1$ to time $t = t_2$ is given by

$$\text{ARC} = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt .$$

Here are two alternate formulas for *ARC LENGTH* :

I.) If a graph is given in *rectangular* form by $y = f(x)$ from $x = a$ to $x = b$ then

$$\text{ARC} = \int_a^b \sqrt{1 + (f'(x))^2} \, dx .$$

II.) If a graph is given in *polar* form by $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ then

$$\text{ARC} = \int_\alpha^\beta \sqrt{(f(\theta))^2 + (f'(\theta))^2} \, d\theta$$