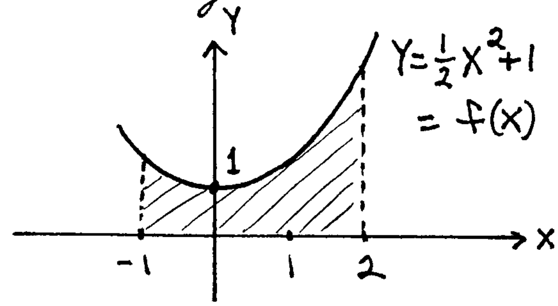
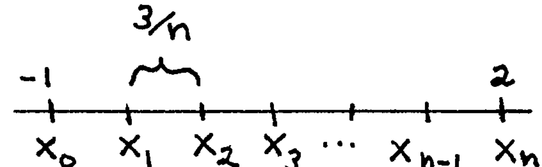
Math 21B
Kouba
Definite Integral

<u>Example</u>: Use equal subintervals and the limit definition for a definite integral to evaluate $\int_{-1}^{2}\left(\frac{1}{2}x^2+1\right)dx$.



Divide the interval $[-1,2]$ into n equal parts, each of length $\frac{3}{n}$. Let the sampling points be the right-hand endpoints of the subintervals:



$x_i = -1 + \frac{3}{n}i$

Then, by limit definition

$$\int_{-1}^{2}\left(\frac{1}{2}x^2+1\right)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,(x_i - x_{i-1})$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{3}{n}i\right)\cdot \frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{1}{2}\left(-1+\frac{3}{n}i\right)^2+1\right]\cdot \frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{9}{2n^2}i^2-\frac{3}{n}i+\frac{3}{2}\right]\cdot \frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{27}{2n^3}i^2-\frac{9}{n^2}i+\frac{9}{2n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{27}{2n^3}\left(\sum_{i=1}^{n}i^2\right)-\frac{9}{n^2}\left(\sum_{i=1}^{n}i\right)+\frac{9}{2n}\left(\sum_{i=1}^{n}1\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{27}{2n^3}\cdot\frac{n(n+1)(2n+1)}{6}-\frac{9}{n^2}\cdot\frac{n(n+1)}{2}+\frac{9}{2n}\cdot n\right]$$

$$= \lim_{n\to\infty}\left[\frac{9}{4}\cdot\frac{2n^2+3n+1}{n^2}-\frac{9}{2}\cdot\frac{n+1}{n}+\frac{9}{2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{9}{4}\cdot\left(2+\frac{3}{n}+\frac{1}{n^2}\right)-\frac{9}{2}\cdot\left(1+\frac{1}{n}\right)+\frac{9}{2}\right]$$

$$= \frac{9}{4}\cdot 2-\frac{9}{2}\cdot 1+\frac{9}{2}$$

$$= \frac{9}{2}.$$