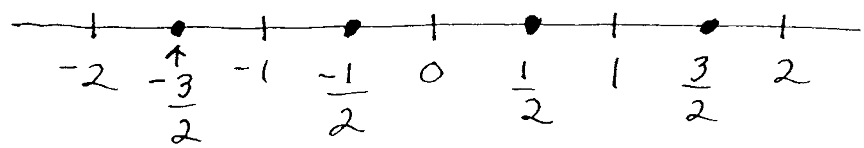
1.) $M = 2^2 e^6 = 4e^6$

2.) $\max\limits_{-1 \le x \le 1} \left| \frac{7}{2x+3} \right| = \left| \frac{7}{2(-1)+3} \right| = 7 = M$

3.) $\max\limits_{-2 \le x \le 3} \left| \frac{x-3}{5-x} \right| \le \left| \frac{(-2)-3}{5-(3)} \right| = \frac{5}{2} = M$

4.) a.)



$h = \frac{2-(-2)}{4} = 1$

$f(x) = \sqrt{x^2+4}$

$$M_4 = (h)\left[f\left(-\tfrac{3}{2}\right) + f\left(-\tfrac{1}{2}\right) + f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{2}\right)\right]$$

$$= (1)\left[\sqrt{\tfrac{9}{4}+4} + \sqrt{\tfrac{1}{4}+4} + \sqrt{\tfrac{1}{4}+4} + \sqrt{\tfrac{9}{4}+4}\right]$$

$$\approx 9.123$$

b.)

$$f'(x) = \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+4}}$$

$$f''(x) = \frac{\sqrt{x^2+4}\,(1) - x \cdot \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x}{x^2+4}$$

$$= \frac{\frac{\sqrt{x^2+4}}{1} - \frac{x^2}{\sqrt{x^2+4}}}{x^2+4} = \frac{x^2+4-x^2}{(x^2+4)^{3/2}} = \frac{4}{(x^2+4)^{3/2}}$$

$$\max_{-2 \le x \le 2} |f''(x)| = \max_{-2 \le x \le 2} \left| \frac{4}{(x^2+4)^{3/2}} \right| \le \frac{4}{4^{3/2}} = \frac{1}{2}$$

$$|E_n| \le \frac{(2-(-2))\left(\frac{2-(-2)}{n}\right)^2}{24} \left\{\frac{1}{2}\right\} = \frac{4}{48} \cdot \frac{16}{n^2} = \frac{4}{3} \cdot \frac{1}{n^2}$$

$$\le 0.0001 \rightarrow n^2 \ge \frac{4}{0.0003} \rightarrow n \ge 115.47$$

choose $n = 116$

5.) a.)

$$0 \qquad 1 \qquad 2 \qquad 3 \qquad 4$$

$$h = \frac{4-0}{4} = 1 \quad , \quad f(x) = \ln(x^2+1)$$

$$T_4 = \frac{h}{2}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right]$$

$$= \frac{1}{2}\left[\ln 1 + 2\ln 2 + 2\ln 5 + 2\ln 10 + \ln 17\right]$$

$$\approx 6.022$$

b.) $$f'(x) = \frac{2x}{x^2+1} \quad , \quad f''(x) = \frac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} \quad ,$$

$$\max_{0 \le x \le 4} |f''(x)| = \max_{0 \le x \le 4} \left|\frac{2-2x^2}{(x^2+1)^2}\right| \le \frac{30}{1} = 30 \quad ,$$

$$|E_n| \le \frac{(4-0)\left(\frac{4-0}{n}\right)^2}{12}\{30\} = \frac{\overset{10}{\cancel{120}}}{\cancel{12}} \cdot \frac{16}{n^2} = \frac{160}{n^2} \le 0.0001$$

$$\longrightarrow n^2 \ge \frac{160}{0.0001} \longrightarrow n \ge \sqrt{\frac{160}{0.0001}} \ge 1265$$

6.) a.)

$$0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

$h = \frac{2-0}{4} = \frac{1}{2}$, $f(x) = \frac{x+1}{x+3}$

$$S_4 = \frac{h}{3}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right]$$

$$= \frac{1}{6}\left[\frac{1}{3} + 4\cdot\frac{3/2}{7/2} + 2\cdot\frac{2}{4} + 4\cdot\frac{5/2}{9/2} + \frac{3}{5}\right]$$

$$= \frac{1}{6}\left[\frac{1}{3} + \frac{12}{7} + 1 + \frac{20}{9} + \frac{3}{5}\right] \approx 0.978$$

b.) $f'(x) = \dfrac{(x+3)(1) - (x+1)(1)}{(x+3)^2} = 2(x+3)^{-2}$

$f''(x) = -4(x+3)^{-3}$, $f'''(x) = 12(x+3)^{-4}$, $f^{(4)}(x) = \dfrac{-48}{(x+3)^5}$,

$$\max_{0 \le x \le 2} |f^{(4)}(x)| = \max_{0 \le x \le 2} \left|\frac{-48}{(x+3)^5}\right| \le \frac{48}{405},$$

$$|E_4| \le \frac{(2-0)\cdot\left(\frac{2-0}{n}\right)^4}{180}\left\{\frac{48}{405}\right\} = \frac{\cancel{2}}{\cancel{180}_{\cancel{90}\,45}} \cdot \frac{\cancel{48}^{16}}{\cancel{405}_{135}} \cdot \frac{\cancel{16}^{8}}{n^4}$$

$$= \frac{16(8)}{45(135)} \cdot \frac{1}{n^4} \le 0.0001 \rightarrow$$

$$n^4 \ge \frac{16(8)}{45(135)(0.0001)} \rightarrow$$

$n \ge 3.8 \longrightarrow n = 4$