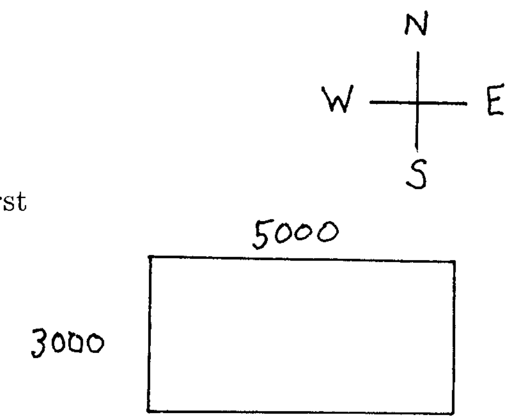
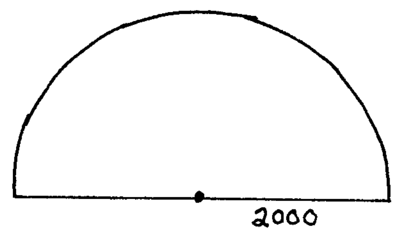
Math 21B
Kouba
Setting Up Definite Integrals Using Local Approximations First



1.) Snow has covered a field 3000 ft. by 5000 ft. The depth of snow x ft. from the west edge of the field is given to be $(16+x)^{1/4}$ ft. Find the total volume of snow on the field.



2.) A semi-circular hay field of radius 2000 m. is infested with grasshoppers. The density of grasshoppers x meters north of the semi-circle's diameter is given to be $(20+(1/10)x)$ grasshoppers/m.^{2} Find the total number of grasshoppers in the field.

3.) A flat plate of variable density is in the shape of an equilateral triangle of edge length 2 ft. It's density x ft. from its base is given to be $\sqrt{x+4}$ kg./ft.^{2} Find the total mass of the plate.

4.) A flat circular plate of uniform density has mass 100 kg. and radius 3 meters. Find the total kinetic energy of the plate if it rotates

a.) 25 times per second around its center.

b.) 25 radians per second around its diameter.

c.) 25 times per second around a line tangent to the edge of the plate.

5.) Find the total kinetic energy of the plate in problem 3.) if it rotates at 50 radians per second around its base.

6.) A sphere of radius 5 ft. has variable density. Its density r ft. from its center is given to be (r^2+3r) lbs./ft.^{3} Find the total weight of the ball. HINT : Assume that the surface area of a ball of radius r is $4\pi r^2$.

7.) You are given a flat plate 5 ft. by 7 ft. Assume that water weighs 62.5 lbs./ft.^{3}

a.) Find the total force (weight) of water on the plate if it lies horizontally (flat) in 10 ft. of water in a swimming pool. HINT : Simply compute the weight of the column of water above the plate.

b.) Find the total force (weight) of water on the plate if it rests on its 5 ft. edge in 10 ft. of water in a swimming pool.

c.) Find the total force (weight) of water on the plate if it rests on its 5 ft. edge and tilts at a 30 degree angle to the bottom in 10 ft. of water in a swimming pool.

8.) Work is defined as force (weight) times distance. If force is measured in lbs. and distance in ft. then the units for work are ft.-lbs. Consider a tank holding 2000 lbs. of water.

a.) If the tank is raised vertically 500 ft., how much work is done in raising the water ?

b.) Assume that the tank is raised vertically 500 ft., but that it begins leaking as soon as the tank is raised. If the tank loses 2 lbs. of water per each foot it is raised, how much work is done in raising the water ?

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ANSWERS To Setting Up Definite Integrals Using Local Approximations First

1.) local estimate : volume = (area) (depth) $\approx (3000)(dx)(16+x)^{1/4}$ ft.^{3};

$$\text{Total Volume} = \int_0^{5000} 3000\ (16+x)^{1/4}\ dx \quad \text{ft.}^3$$

2.) local estimate : number of grasshoppers = (area) (density of grasshoppers)
$\approx (2\sqrt{2000^2-x^2})(dx)(20+(1/10)x)$ grasshoppers ;

$$\text{Total Grasshoppers} = \int_0^{2000} (20+(1/10)x)(2\sqrt{2000^2-x^2})\ dx \quad \text{grasshoppers.}$$

3.) local estimate : mass = (area) (density) $\approx (2)\left(\dfrac{\sqrt{3}-x}{\sqrt{3}}\right)(dx)\sqrt{x+4}$ kg. ;

$$\text{Total Mass} = \int_0^{\sqrt{3}} 2\left(\frac{\sqrt{3}-x}{\sqrt{3}}\right)\sqrt{x+4}\ dx \quad \text{kg.}$$

4.) uniform density $= \dfrac{mass}{area} = \dfrac{100\ kg.}{9\pi\ m.^2} = \dfrac{100}{9\pi}$ kg./m.^{2} ;

a.) local estimate : mass = (area) (density) $\approx (2\pi r)(dr)\left(\dfrac{100}{9\pi}\right)$ kg. ;

$$\text{velocity} \approx \frac{(2\pi r)\ m.}{circle}\ \frac{25\ circles}{sec.} = 50\pi r\ \frac{m.}{sec.}\ ;$$

$$\text{kinetic energy} = \frac{1}{2}\ (\text{mass})(\text{velocity})^2 = \approx \frac{1}{2}\left(\frac{200}{9}r(dr)\right)(50\pi r)^2 \quad \text{joules ;}$$

$$\text{Total K.E.} = \int_0^3 \frac{1}{2}\left(\frac{200}{9}r\right)(50\pi r)^2\ dr \quad \text{joules.}$$

b.) local estimate : mass = (area) (density) $\approx \left(2\sqrt{9-x^2}\right)(dx)\left(\dfrac{100}{9\pi}\right)$ kg. ;

$$\text{velocity} \approx \frac{(2\pi x)\ m.}{circle}\ \frac{1\ circles}{2\pi\ rad.}\ \frac{25\ rad.}{sec.} = 25x\ \frac{m.}{sec.}\ ;$$

$$\text{kinetic energy} = \frac{1}{2}\ (\text{mass})(\text{velocity})^2 = \approx \frac{1}{2}\left(\frac{200}{9\pi}\sqrt{9-x^2}(dx)\right)(25x)^2 \quad \text{joules ;}$$

$$\text{Total K.E.} = 2\int_0^3 \frac{1}{2}\left(\frac{200}{9\pi}\sqrt{9-x^2}\right)(25x)^2\ dx \quad \text{joules} \qquad \text{OR}$$

$$\text{Total K.E.} = \int_{-3}^3 \frac{1}{2}\left(\frac{200}{9\pi}\sqrt{9-x^2}\right)(25x)^2\ dx \quad \text{joules.}$$

c.) local estimate : mass = (area) (density) $\approx \left(2\sqrt{9-(x-3)^2}\right)(dx)\left(\dfrac{100}{9\pi}\right)$ kg. ;

$$\text{velocity} \approx \frac{(2\pi x)\ m.}{circle}\ \frac{25\ circles}{sec.} = 50\pi x\ \frac{m.}{sec.}\ ;$$

$$\text{kinetic energy} = \frac{1}{2}\ (\text{mass})(\text{velocity})^2 = \approx \frac{1}{2}\left(\frac{200}{9\pi}\sqrt{9-(x-3)^2(dx)}\right)(50\pi x)^2 \quad \text{joules}$$

;

$$\text{Total K.E.} = \int_0^6 \frac{1}{2}\Big(\frac{200}{9\pi}\sqrt{9-(x-3)^2}\Big)(50\pi x)^2 \ dx \quad \text{joules.}$$

5.) local estimate : mass = (area) (density) $\approx (2)\Big(\frac{\sqrt{3}-x}{\sqrt{3}}\Big)(dx)\sqrt{x+4}$ kg. ;

$$\text{velocity} \approx \frac{(2\pi x)\ ft.}{circle}\ \frac{1\ circles}{2\pi\ rad.}\ \frac{50\ rad.}{sec.} = 50x\ \frac{ft.}{sec.}\ ;$$

kinetic energy $= \frac{1}{2}$ (mass)(velocity)$^2 \approx \frac{1}{2}\Big[(2)\Big(\frac{\sqrt{3}-x}{\sqrt{3}}\Big)(dx)\sqrt{x+4}\Big](50x)^2$ kg. ft.^{2}/sec.^{2} ;

$$\text{Total K.E.} = \int_0^{\sqrt{3}} \frac{1}{2}\Big[(2)\Big(\frac{\sqrt{3}-x}{\sqrt{3}}\Big)\sqrt{x+4}\Big](50x)^2 \ dx \quad \text{kg. ft.}^2/\text{sec.}^2$$

6.) local estimate : weight = (density) (volume) $\approx (r^2+3r)(4\pi r^2)(dr)$ lbs. ;

$$\text{Total Weight} = \int_0^5 (r^2+3r)(4\pi r^2) \ dr \quad \text{lbs.}$$

7.) a.) force = (volume)(density of water) = (area)(depth)(density of water)
= (5)(7)(10)(62.5) = 21,875 lbs.

b.) local estimate : force = (area)(depth)(density of water) $\approx (5)(dx)(10-x)(62.5)$ lbs.

$$\text{Total Force} = \int_0^7 5(62.5)(10-x) \ dx \quad \text{lbs.}$$

c.) local estimate : force = (area)(depth)(density of water) $\approx (5)(2dx)(10-x)(62.5)$ lbs.

$$\text{Total Force} = \int_0^{7/2} 10(62.5)(10-x) \ dx \quad \text{lbs.}$$

8.) a.) work = (force)(distance) = (2000)(500) = 1,000,000 ft.-lbs.

b.) local estimate : work = (force)(distance) $\approx (2000-2x)(dx)$ ft.-lbs. ;

$$\text{Total Work} = \int_0^{500} (2000-2x) \ dx \quad \text{ft.-lbs.}$$