Area of Surface of Revolution

Consider the graph of \( y = f(x) \) for \( a \leq x \leq b \).

Create a Surface of Revolution by revolving the graph about the \( x \)-axis.
Consider a thin, circular slice of the surface at \( x \) of width \( ds \):

We will assume that

\[
(ds)^2 = (dx)^2 + (dy)^2 \quad \Rightarrow 
\]

\[
ds = \sqrt{(dx)^2 + (dy)^2} \; ; \; \text{the slice has area approximately} 
\]

\[
\text{(length)(width)} = 2\pi f(x) \cdot ds 
\]

\[
= 2\pi f(x) \cdot \sqrt{(dx)^2 + (dy)^2} 
\]

\[
= 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx 
\]

so total surface area is
\[\text{Area} = 2\pi \int_{a}^{b} f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx\]

Assume that the graph of \(x = g(y)\) for \(c \leq y \leq d\) is revolved about the y-axis. In a similar fashion, it can be shown that the total surface area is

\[\text{Area} = 2\pi \int_{c}^{d} g(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy\]