Math 21B
Kouba
Discussion Sheet 9

1.) Assume that $f''(x) = x^2 e^{3x}$. Find a number M so that $\max_{0 \le x \le 2} |f''(x)| \le M$.

2.) Assume that $f''(x) = \dfrac{7}{2x+3}$. Find a number M so that $\max_{-1 \le x \le 1} |f''(x)| \le M$.

3.) Assume that $f^{(4)}(x) = \dfrac{x-3}{5-x}$. Find a number M so that $\max_{-2 \le x \le 3} |f^{(4)}(x)| \le M$.

4.) Consider the definite integral $\displaystyle\int_0^4 \ln(x^2+1)\, dx$.

a.) Find T_4 , the Trapezoidal Estimate using $n = 4$.
b.) What should n be in order that the Trapezoidal Estimate T_n estimate the exact value of this definte integral with absolute error at most 0.0001 ?

5.) Consider the definite integral $\displaystyle\int_0^2 \frac{x+1}{x+3}\, dx$.

a.) Find S_4 , the Simpson Estimate using $n = 4$.
b.) What should n be in order that the Simpson Estimate S_n estimate the exact value of this definte integral with absolute error at most 0.0001 ?

6.) Consider the region R lying below the graph of $y = \dfrac{1}{x}$ and above the x-axis on the interval $[1, \infty)$.
a.) Determine if R has finite or infinite area.
b.) Form a solid by revolving R about the x-axis. Determine if the resulting volume is finite or infinite.
c.) Form a solid by revolving R about the y-axis. Determine if the resulting volume is finite or infinite.

7.) Compute the following improper integrals.

a.) $\displaystyle\int_1^\infty \frac{1}{x(x+4)}\, dx$ b.) $\displaystyle\int_{-\infty}^0 e^{3x}\, dx$ c.) $\displaystyle\int_{-1}^\infty \frac{1}{\sqrt{x+1}}\, dx$

d.) $\displaystyle\int_{-\infty}^{\sqrt{3}} \frac{1}{x^2+9}\, dx$ e.) $\displaystyle\int_1^{e^2+1} \frac{7}{x-1}\, dx$ f.) $\displaystyle\int_0^e x^2 \ln x\, dx$

g.) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^2}\, dx$ h.) $\displaystyle\int_0^5 \frac{1}{\sqrt{25-x^2}}\, dx$ i.) $\displaystyle\int_1^\infty \frac{24}{2x^2+5x+2}\, dx$

j.) $\displaystyle\int_0^5 \frac{8x}{x^2-9}\, dx$ k.) $\displaystyle\int_{-\infty}^\infty x^2 e^{x^3}\, dx$ l.) $\displaystyle\int_0^{\pi/2} \csc x \cot x\, dx$

m.) $\displaystyle\int_0^\infty x e^{-5x}\, dx$ n.) $\displaystyle\int_0^\infty \frac{1}{x^2}\, dx$ o.) $\displaystyle\int_0^1 \ln x\, dx$ p.) $\displaystyle\int_0^\infty \frac{e^{-1/x}}{x^2}\, dx$

8.) Use the Comparison Test to show that the following improper integral converges, i.e., is finite : $$\int_1^{\infty} \frac{1}{\sqrt{x^3+16}}\ dx$$

9.) Use the Comparison Test to show that the following improper integral diverges, i.e., is infinite : $$\int_2^{\infty} \frac{x+4}{\sqrt{x^3+16}}\ dx$$

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“ What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises.”– G. Lichtenberg

“ I hear and I forget. I see and I remember. I do and I understand.”– Chinese Proverb