Let $R$ be a flat region in two-dimensional space and let $\delta(P)$ be the density of the region at point $P = (x, y)$.

1.) AREA: $\int_R 1 \, dA$ represents the area of region $R$.

2.) AVERAGE VALUE: \( \frac{1}{\text{Area of } R} \int_R f(x, y) \, dA \) represents the average value of function $z = f(x, y)$ over region $R$.

3.) MASS: $\int_R \delta(P) \, dA$ represents the mass of region $R$.

4.) VOLUME: $\int_R f(P) \, dA$ represents the volume of the solid region defined on region $R$ with height $f(P)$ at point $P$.

5.) MOMENT:
   a.) $\int_R (x - a) \delta(P) \, dA$ represents the moment of region $R$ about the vertical line $x = a$.
   b.) $\int_R (y - b) \delta(P) \, dA$ represents the moment of region $R$ about the horizontal line $y = b$.

6.) CENTER OF MASS, $(\bar{x}, \bar{y})$:
   a.) $\bar{x} = \frac{\int_R x \delta(P) \, dA}{\int_R \delta(P) \, dA}$ represents the $x$-coordinate of the center of mass of region $R$.
   b.) $\bar{y} = \frac{\int_R y \delta(P) \, dA}{\int_R \delta(P) \, dA}$ represents the $y$-coordinate of the center of mass of region $R$.

7.) CENTROID, $(\bar{x}, \bar{y})$:
   a.) $\bar{x} = \frac{\int_R x \, dA}{\int_R 1 \, dA}$ represents the $x$-coordinate of the centroid of region $R$.
   b.) $\bar{y} = \frac{\int_R y \, dA}{\int_R 1 \, dA}$ represents the $y$-coordinate of the centroid of region $R$.

NOTE: The formulas for centroid follow immediately from the formulas for center of mass by letting density $\delta(P) = 1$.

8.) MOMENT OF INERTIA: $\int_R (\text{distance})^2 \delta(P) \, dA$ represents the moment of inertia of region $R$, where distance refers to the distance from point $P = (x, y)$ in region $R$ to either a point or axis (line) of rotation.