Math 21C
Kouba
Discussion Sheet 2

1.) Consider the graph of $y = \ln(x-1)$ in the xy-plane. Find an equation for the surface created by revolving this graph about the

a.) x-axis . b.) y-axis .

2.) Sketch the domain of each function.

a.) $f(x,y) = \ln(1+x^2+y^2)$ b.) $f(x,y) = \ln(1+x+y)$

c.) $f(x,y) = \dfrac{1}{\sqrt{9-x^2-y^2}}$ d.) $f(x,y) = \sqrt{(x^2-4)(y^2-1)}$

3.) Evaluate the following limits or determine that the limit does not exist.

a.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2-4}{x+y+2}$ b.) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}$

c.) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2}$ d.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

e.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3}{x^3+y^3}$ f.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ g.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

4.) Compute z_x and z_y for each of the following functions.

a.) $z = xy^2 + \ln x + e^y + 5$ b.) $z = xe^{2y}\arctan x$ c.) $z = \sqrt{x-y^2}$

d.) $z = \dfrac{x^3}{y^2} + \sin(xy)$ e.) $z = \dfrac{x+4}{x^2+y^2}$ f.) $z = \{e^{x^2y} + \tan(3y+4x)\}^5$

f.) $z = y^{1+x^3}$

5.) Show that $z = \ln(1+x^2+y^2)$ satisfies the equation $z_{xy} + z_x z_y = 0$.

6.) Verify that $w_{xy} = w_{yx}$ for $w = y + \dfrac{x}{y}$.

7.) Determine functions z whose partial derivatives are given, or state that this is impossible.

a.) $z_x = 2x$ and $z_y = 3y^2+1$ b.) $z_x = xy^2 - y$ and $z_y = x^2y - x$

c.) $z_x = e^x y - 1$ and $z_y = e^x - x$

d.) $z_x = ye^x\cos(xy) + e^x\sin(xy)$ and $z_y = xe^x\cos(xy) + 1$

8.) Consider the function $f(x,y) = \begin{cases} \dfrac{\sin(x^3+y)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 \quad , & \text{if } (x,y) = (0,0) \ . \end{cases}$

a.) Determine $f_x(x,y)$ when $(x,y) \neq (0,0)$.

b.) Determine $f_x(0,0)$. c.) Determine $f_y(0,0)$.

9.) Plane A, parallel to the xz-plane, and plane B, parallel to the yz-plane, pass through the surface determined by the equation $z = xy^2 - x^3 + 7$. Both planes include the point $(1,0,6)$, which lies on the surface.

a.) Determine the slope of the line tangent to the surface at the point $(1,0,6)$ if the line lies in

i.) plane A.

ii.) plane B.

b.) Determine an equation of the plane tangent to the surface at the point $(1,0,6)$.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

10.) A 12 ft. by 30 ft. room has a 12 ft. ceiling. In the middle of one end wall, one foot above the floor, is a spider. The spider wants to capture a fly in the middle of the opposite end wall, one foot below the ceiling. What is the length of the shortest path the spider can walk (no spider webs allowed) in order to reach the fly ?