Math 21C
Kouba
Discussion Sheet 3

1.) Compute z_x and z_y for each of the following functions.
a.) $z = x^3y + y^4 - 2x + 5$ b.) $z = f(x) + g(y)$ c.) $z = f(x^3) + g(4y)$
d.) $z = f(x^2 + y^3) + g(xy^2)$ e.) $y^2 + z^2 + \sin(xz) = 4$
f.) $z = f(u, v)$ where $u = \ln(x - y)$ and $v = e^{xy}$

2.) Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ if $w = f(4t^2 - 3s)$ and $f'(x) = \ln x$.

3.) Assume that f is differentiable function of one variable with $z = xf(xy)$. Show that $xz_x - yz_y = z$.

4.) Assume that f and g are twice differentiable functions of one variable. Show that $u = f(x + at) + g(x - at)$ satisfies $a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, where a is a constant.

5.) Find and classify critical points as determining relative maximums, relative minimums, or saddle points.
a.) $z = 3x^2 - 6xy + y^2 + 12x - 16y + 1$
b.) $z = x^2y - x^2 - 2y^2$
c.) $z = x^2 - 8\ln(xy) + y^2$
d.) $z = 3x^2y - 6x^2 + y^3 - 6y^2$

6.) Determine the absolute extrema for each function on the indicated region.
a.) $f(x, y) = 2x + 4y + 12$ on
i.) the triangle with vertices $(0, 0), (0, 3)$, and $(3, 0)$.
ii.) the circle $x^2 + y^2 = 4$.
b.) $f(x, y) = xy - x - 3y$ on the triangle with vertices $(0, 0), (0, 4)$, and $(5, 0)$.
c.) $f(x, y) = x^2 - 3y^2 - 2x + 6y$ on the square with vertices $(0, 0), (0, 2), (2, 0)$ and $(2, 2)$.

7.) Find the point on the plane $x + 2y + 3z = 6$ nearest the origin.

8.) Determine the minimum surface area of a closed rectangular box with volume 8 ft.^{3}

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

9.) Divide the figure into four equal parts, each one of the same size and shape.