

Math 21C  
Kouba  
Discussion Sheet 6

1.) Let  $R$  be the solid region bounded by the surfaces  $z = \sqrt{4 - x^2 - y^2}$  and  $z = 0$ . SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid

- using rectangular coordinates.
- using cylindrical coordinates.
- using spherical coordinates.

2.) Let  $R$  be the solid region bounded by the surfaces  $z = \sqrt{x^2 + y^2}$  and  $z = \sqrt{18 - x^2 - y^2}$ . SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid

- using rectangular coordinates.
- using cylindrical coordinates.
- using spherical coordinates.

3.) Let  $R$  be the solid region inside the surface  $x^2 + y^2 = 4$  and bounded by the surfaces  $z = 0$  and  $z = \sqrt{9 - x^2 - y^2}$ . SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid

- using rectangular coordinates.
- using cylindrical coordinates.
- using spherical coordinates.

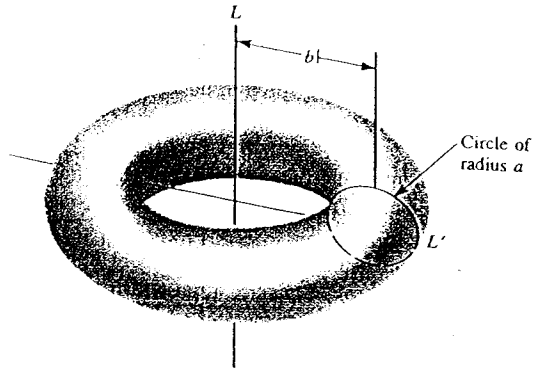
4.) Consider the UFO bounded by the surfaces  $z = x^2 + y^2$  and  $z = 8 - x^2 - y^2$ . The density of the UFO at point  $P = (x, y, z)$  is given by the square of the distance from  $P$  to the  $z$ -axis. SET UP BUT DO NOT EVALUATE triple integrals in all three coordinate systems which represent the UFO's

- average density.
- total mass.
- total volume.
- $z$ -coordinate of the centroid.
- $z$ -coordinate of the center of mass.
- moment of inertia about
  - the  $z$ -axis.
  - the  $x$ -axis.

5.) Convert the following integral to rectangular coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\cos \phi} \rho^3 \sin \theta \sin^2 \phi \, d\rho \, d\phi \, d\theta$$

6.) SET UP BUT DO NOT EVALUATE a triple integral which represents the volume of the given doughnut (torus).



7.) Consider the solid region  $R$  bounded by the surfaces  $z = x^2 + y^2$  and  $z = 2y$ . Describe  $R$  using spherical coordinates.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

8.) The camp cook wants to measure exactly four ounces of vinegar out of a jug, but has only a five-ounce container and a three-ounce container. How can the cook accomplish the task ?