Math 21C
Kouba
Discussion Sheet 9

1.) Determine convergence or divergence of each series using the test indicated. I suggest that you read all of the assumptions and conclusions for each test from the handout I gave last time as you do each problem.

a.) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n-1}}{(n+1)!}$ (Use the ratio test.)

b.) $\displaystyle\sum_{n=1}^{\infty} \left(1.01 - \frac{5}{n^3}\right)^n$ (Use the root test.)

c.) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{7}{n^2+3}$ (Use the alternating series test.)

d.) $1 + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \frac{1}{3^6} + \frac{1}{3^7} - \frac{1}{3^8} + \cdots$ (Use the absolute convergence test.)

e.) $\displaystyle\sum_{n=0}^{\infty} (-1)^n (1/10)$ (Use the sequence of partial sums test.)

f.) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^3+8}}$ (Use the limit comparison test.)

2.) Use a geometric series to convert the decimal number 0.777777777... to a fraction.

3.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

a.) Use equation (*) to determine between which two numbers the partial sum $S_{50} = \displaystyle\sum_{i=1}^{50} \frac{1}{i}$ lies.

b.) What should n be in order that the partial sum $S_n = \displaystyle\sum_{i=1}^{n} \frac{1}{i}$ be at least 20 ?

c.) What is the largest value of n for which the partial sum $S_n = \displaystyle\sum_{i=1}^{n} \frac{1}{i}$ does not exceed 50 ?

4.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

a.) Compute the partial sum $S_5 = \displaystyle\sum_{i=1}^{5} \frac{1}{i^3}$. Use (*)(*) to estimate the resulting error.

b.) What should n be in order that the partial sum $S_n = \sum_{i=1}^{n} \frac{1}{i^3}$ estimate the exact value of the series with error at most 0.0001 ?

5.) The alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3}$ converges.

a.) Compute the partial sum $S_5 = \sum_{i=1}^{5} (-1)^{i+1} \frac{1}{i^3}$. Estimate the resulting absolute error. Between what two numbers does the exact value of the series lie ?

b.) What should n be in order that the partial sum $S_n = \sum_{i=1}^{n} (-1)^{i+1} \frac{1}{i^3}$ estimate the exact value of the series with absolute error at most 0.0001 ?

"What is important is to keep learning, to enjoy challenge, and to tolerate ambiguity. In the end there are no certain answers." – Martina Horner