Math 21C
Kouba
Optional Power Series Problems

1.) Use the known Maclaurin series for e^x, $\sin x$, $\cos x$, $\dfrac{1}{1-x}$, and $\ln(1+x)$ together with standard shortcuts (substitution, polynomial multiplication, polynomial division, differentiation, and integration) to find the first three nonzero terms of the Maclaurin series for each of the following functions.

a.) $e^x \ln(1+x)$ b.) $\dfrac{2x+x^2}{\ln(1-x)}$ c.) $(\sin(x/3))^2$ d.) $\dfrac{x^2}{e^{2x}-\cos x}$

e.) $\dfrac{6}{(1-x)^4}$ (HINT: Start with $\dfrac{1}{1-x}$ and use differentiation.)

f.) $x^3 \arctan(x^2)$ (HINT: Start with $\dfrac{1}{1+x^2}$ and use integration.)

2.) Use the absolute error for an alternating series, $|R_n|$, to estimate the exact value of each integral, which cannot be solved using the Fundamental Theorem of Calculus, with error at most 0.0001.

a.) $\displaystyle\int_0^1 \cos(x^4)\,dx$ b.) $\displaystyle\int_0^{1/2} e^{-x^2}\,dx$

3.) Use the absolute Lagrange error (remainder) for a Taylor series, $|R(x;a)| = \left|\dfrac{f^{(n+1)}(c_n)(x-a)^{n+1}}{(n+1)!}\right|$, where c_n is between x and a, to estimate the exact value of each number with error at most 0.0001.

a.) $e^{0.7}$ b.) $\sin(1.1)$ c.) $\ln(0.8)$

4.) Write each Maclaurin series as a simple function.

a.) $\dfrac{x^3}{3!} - \dfrac{x^5}{5!} + \dfrac{x^7}{7!} - \dfrac{x^9}{9!} + \cdots$

b.) $5x - \dfrac{5^3x^3}{3!} + \dfrac{5^5x^5}{5!} - \dfrac{5^7x^7}{7!} + \cdots$

c.) $2 + 3\cdot 2\cdot x + 4\cdot 3\cdot x^2 + 5\cdot 4\cdot x^3 + 6\cdot 5\cdot x^4 + \cdots$

d.) $1 - \dfrac{x}{2!} + \dfrac{x^2}{4!} - \dfrac{x^3}{6!} + \dfrac{x^4}{8!} - \cdots$

e.) $x^3 - \dfrac{x^6}{2} + \dfrac{x^9}{3} - \dfrac{x^{12}}{4} + \dfrac{x^{15}}{5} - \cdots$