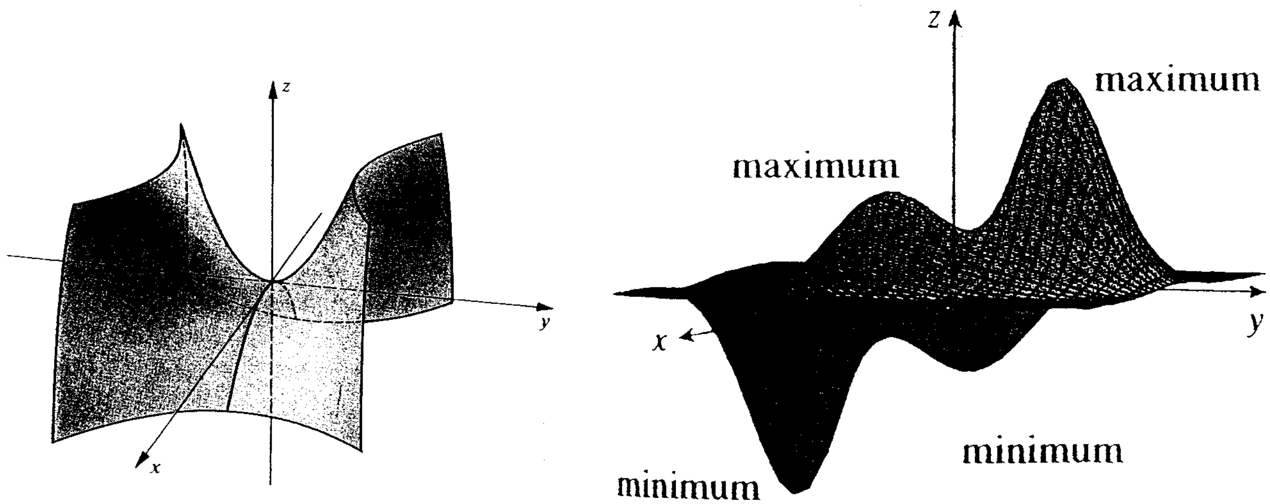
Math 21C
Kouba
The Second Derivative Test for Relative Extrema in Three Dimensional Space

We seek to find the relative maximum and relative minimum values of surfaces in three-dimensional space given by the function $z = f(x, y)$.



SECOND DERIVATIVE TEST :

1.) First compute the partial derivatives $\dfrac{\partial f}{\partial x} = f_x$ and $\dfrac{\partial f}{\partial y} = f_y$. Then find all points (a, b) which satisfy

$$f_x = 0 \text{ and } f_y = 0.$$

These points (a, b) are called critical points.

2.) Determine the partial derivatives f_{xx}, f_{yy}, and f_{xy}. For each of the critical points compute the discriminant

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2 \ .$$

3.) a.) If $D > 0$ and $f_{xx} > 0$, then f has a relative minimum value at (a, b).

b.) If $D > 0$ and $f_{xx} < 0$, then f has a relative maximum value at (a, b).

c.) If $D < 0$, then f has a saddle point at (a, b). In other words, at the point (a, b) there is a path along which $z = f(a, b)$ appears to be a maximum and another path along which $z = f(a, b)$ appears to be a minimum.

d.) For all other cases (for example, if $D = 0$) this test is INCONCLUSIVE. This means other methods must be used to determine if the critical point determines a maximum value, minimum value, or saddle point.