Math 21C DHC
Kouba
Discussion Sheet 8

1.) Determine if the following series converge or diverge.

a.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n!)^2}{(2n)!}$ b.) $\displaystyle\sum_{n=2}^{\infty}\frac{(n!)^2}{2^{n^2}}$ c.) $\displaystyle\sum_{n=3}^{\infty}\frac{1}{(\ln n)^{1/n}}$

d.) $\displaystyle\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}$ e.) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n}+(-1)^n}$ f.) $\displaystyle\sum_{n=3}^{\infty}\frac{(-1)^{n+1}}{\ln(1+1/n)}$

g.) $\displaystyle\sum_{n=1}^{\infty}\ln(n\sin(1/n))$ h.) $\displaystyle\sum_{n=2}^{\infty}(\sqrt{n^2+1}-n)$ i.) $\displaystyle\sum_{n=3}^{\infty}(-1)^n\int_n^{n+1}\frac{e^{-x}}{x}\,dx$

2.) a.) Show that $\displaystyle\lim_{n\to\infty}\sum_{k=n}^{2n}\frac{1}{k}=\ln 2$. HINT : Use rectangles, a definite integral, and the Squeeze Principle.

b.) Assume that p and q are positive integers with $p\ge q\ge 1$. Show that $\displaystyle\lim_{n\to\infty}\sum_{k=qn}^{pn}\frac{1}{k}=\ln(p/q)$.

3.) Determine the interval of convergence for each power series.

a.) $\displaystyle\sum_{n=1}^{\infty}\frac{3^{\sqrt{n}}}{n}x^n$ b.) $\displaystyle\sum_{n=4}^{\infty}(-1)^n\frac{(x-1)^n}{(n+1)2^n}$ c.) $\displaystyle\sum_{n=1}^{\infty}[1-(-2)^n]x^n$

d.) $\displaystyle\sum_{n=1}^{\infty}\frac{(x+1)^n}{2^n+3^n}$ e.) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{1}{n}+\frac{2^n}{n^2}\right)x^n$

4.) Write out the first four nonzero terms of the Maclaurin series for each function.

a.) $x^4e^{2x}\sin(3x)$ b.) $\dfrac{(x-1+e^{-x})\cos(x^2)}{\sin^2(2x)}$ c.) $\dfrac{x\ln(1+x)+\arctan x}{(x-1)\cos x}$

5.) Use a Maclaurin series to estimate the value of $\displaystyle\int_0^{0.75}x^2\cos(x^2)\,dx$ with absolute error at most 0.0001. HINT : Use an alternating series.

6.) Use a Maclaurin series to estimate the value of $\displaystyle\int_0^{0.9}\frac{\sin(x^3)}{x}\,dx$ with absolute error at most 0.0001. HINT : Use an alternating series.

7.) Use a Maclaurin series to estimate the value of $e^{0.3}$ with absolute error at most 0.0001. HINT : Use the Lagrange Form of the Taylor Remainder.

8.) Use a Maclaurin series to estimate the value of $\ln(0.8)$ with absolute error at most 0.0001. HINT : Use the function $\ln(1+x)$ with $x=-0.2$ and the Lagrange Form of the Taylor Remainder.