Math 21C
Kouba
Problems Using (*) and (*)(*) from the Integral Test Handout

1.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Use (*) to put a lower and an upper bound on the partial sum

a.) s_{10}, the sum of the first 10 terms of this series.
b.) s_{1000}, the sum of the first 1000 terms of this series.
c.) $s_{1,000,000}$, the sum of the first 1,000,000 terms of this series.

2.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

a.) Compute the partial sum $s_{10} = \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots + \dfrac{1}{10^2} = \displaystyle\sum_{i=1}^{10} \frac{1}{i^2}$.

b.) Use (*)(*) to put a lower and an upper bound on the error (remainder) $R_{10} = \dfrac{1}{11^2} + \dfrac{1}{12^2} + \dfrac{1}{13^2} + \cdots$ for the partial sum s_{10}.

c.) Use (*)(*) to put a lower and an upper bound on the error (remainder) $R_{100} = \dfrac{1}{101^2} + \dfrac{1}{102^2} + \dfrac{1}{103^2} + \cdots$ for the partial sum s_{100}.

d.) What should n be so that the partial sum $s_n = \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots + \dfrac{1}{n^2} = \displaystyle\sum_{i=1}^{n} \frac{1}{i^2}$ estimates the exact value of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ with an error R_n of at most 0.0001 ?

3.) The series $\displaystyle\sum_{n=1}^{\infty} (2/3)^{n-1}$ converges.

a.) Compute the partial sum $s_{10} = 1 + (2/3) + (2/3)^2 + \cdots + (2/3)^9 = \displaystyle\sum_{i=1}^{10} (2/3)^{i-1}$.

b.) Use (*)(*) to put a lower and an upper bound on the error (remainder) $R_{10} = (2/3)^{10} + (2/3)^{11} + (2/3)^{12} + (2/3)^{13} + \cdots$.

c.) Compute the exact value of $R_{10} = (2/3)^{10} + (2/3)^{11} + (2/3)^{12} + (2/3)^{13} + \cdots$.

d.) What should n be so that the partial sum $s_n = 1 + (2/3) + (2/3)^2 + \cdots + (2/3)^{n-1} = \displaystyle\sum_{i=1}^{n}(2/3)^{i-1}$ estimates the exact value of $\displaystyle\sum_{n=1}^{\infty}(2/3)^{n-1}$ with an error R_n of at most 0.0001 ?

e.) What is the exact value of the series $\displaystyle\sum_{n=1}^{\infty} (2/3)^{n-1}$?