Math 21C
Kouba
Subtle Facts About Infinite Series (Given Without Proof)

1.) a.) The series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $\sum_{n=k}^{\infty} a_n$ converges, i.e., addition or removal of a finite number of terms in a convergent series does not affect convergence.

b.) The series $\sum_{n=1}^{\infty} a_n$ diverges if and only if the series $\sum_{n=k}^{\infty} a_n$ diverges, i.e., addition or removal of a finite number of terms in a divergent series does not affect divergence.

2.) a.) Let c be a nonzero constant. The series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $c \cdot \sum_{n=k}^{\infty} a_n$ converges, i.e., scalar multiplication of a convergent series does not affect convergence.

b.) Let c be a nonzero constant. The series $\sum_{n=1}^{\infty} a_n$ diverges if and only if the series $c \cdot \sum_{n=k}^{\infty} a_n$ diverges, i.e., scalar multiplication of a divergent series does not affect divergence.

3.) a.) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} (a_n + b_n)$ converges.

b.) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} (a_n + b_n)$ diverges.

c.) If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} (a_n + b_n)$ may diverge or it may converge. (HINT: Consider $a_n = \frac{1}{n}$ and $b_n = \frac{2}{n}$. Consider $a_n = \frac{1}{n}$ and $b_n = \frac{-1}{n+1}$.)

4.) The Nth Term Test (Divergence Test) states that if $\lim_{n \to \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. This is equivalent to the statement if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$. (An if/then statement and its contrapositive are logically equivalent.)

5.) When using the formula $\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + r^4 + \cdots = \frac{1}{1-r}$ for a *convergent* Geometric Series, it is necessary that the first term in the series be 1.

6.) When using the Integral Test to test $\sum_{n=1}^{\infty} a_n$ for convergence or divergence, the integral $\int_1^{\infty} f(x)\, dx$ can be replaced with $\int_k^{\infty} f(x)\, dx$ and not change convergence or divergence. However, if equation (*) or (**) from the Integral Test class handout is being used, then $\sum_{n=k}^{\infty} a_n$ and $\int_k^{\infty} f(x)\, dx$ must both begin with the same value of k.