Math 21C
Kouba
Surfaces of Revolution

PROBLEM: Consider the two-dimensional graph \( G \) of any equation in two variables, i.e., consider \( G \) to be a graph in the \( xy \)-plane, the \( yz \)-plane, or the the \( xz \)-plane. Create a surface of revolution in three-dimensional space by revolving \( G \) around an axis (line) \( L \). (Line \( L \) may be vertical, horizontal, or tilted.) We want to determine an equation for this surface using an arbitrary point \( (x, y, z) \) on the surface.

SOLUTION:

*Step 1.* Select a random point \( P = (x, y, z) \) on the three-dimensional surface. The goal is to use these variables to write an equation which represents this surface.

*Step 2.* Determine the point \( Q \), which

a.) depends on point \( P \),

b.) lies on the axis of revolution \( L \),

and

c.) is nearest point \( P \).

*Step 3.* Determine the point \( R \), which

a.) depends on point \( Q \),

b.) lies on the original graph \( G \),

and so that

c.) points \( P, Q, \) and \( R \) are now part of a *cross-sectional circle* with \( Q \) at the center and segments \( PQ \) and \( QR \) forming radii of the circle.

*Step 4.* Use the distance formula to compute the lengths of \( PQ \) and \( QR \). The equation can now be determined by setting

\[
\text{length } PQ = \text{length } QR .
\]