Math 21C
Kouba
Surfaces of Revolution

PROBLEM : Consider the two-dimensional graph G of any equation in two variables, i.e., consider G to be a graph in the xy-plane, the yz-plane, or the the xz-plane. Create a surface of revolution in three-dimensional space by revolving G around an axis (line) L. (Line L may be vertical, horizontal, or tilted.) We want to determine an equation for this surface using an arbitrary point (x, y, z) on the surface.

SOLUTION :

Step 1. Select a random point $P = (x, y, z)$ on the three-dimensional surface. The goal is to use these variables to write an equation which represents this surface.

Step 2. Determine the point Q, which

a.) depends on point P,
b.) lies on the axis of revolution L,

and

c.) is nearest point P .

Step 3. Determine the point R, which

a.) depends on point Q,
b.) lies on the original graph G,

and so that

c.) points P, Q, and R are now part of a *cross-sectional circle* with Q at the center and segments PQ and QR forming radii of the circle.

Step 4. Use the distance formula to compute the lengths of PQ and QR. The equation can now be determined by setting

$$\text{length } PQ = \text{length } QR \quad .$$