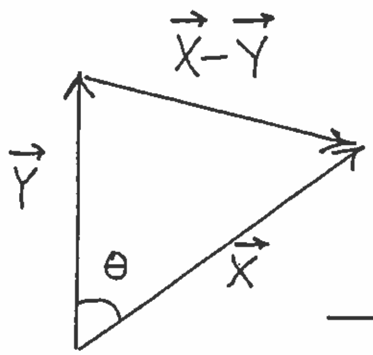
Math 21C
Kouba
An Alternate Formula
for Dot Product

Let $\vec{X}$ and $\vec{Y}$ be vectors and consider the triangle formed by $\vec{X}$, $\vec{Y}$ and $\vec{X}-\vec{Y}$. Let θ be the angle between $\vec{X}$ and $\vec{Y}$. By the Law of Cosines



$$|\vec{X}-\vec{Y}|^2 = |\vec{X}|^2 + |\vec{Y}|^2 - 2|\vec{X}||\vec{Y}|\cos\theta$$

$$\rightarrow (\vec{X}-\vec{Y})\cdot(\vec{X}-\vec{Y}) = |\vec{X}|^2 + |\vec{Y}|^2 - 2|\vec{X}||\vec{Y}|\cos\theta$$

$$\rightarrow \vec{X}\cdot\vec{X} - \vec{X}\cdot\vec{Y} - \vec{Y}\cdot\vec{X} + \vec{Y}\cdot\vec{Y} = |\vec{X}|^2 + |\vec{Y}|^2 - 2|\vec{X}||\vec{Y}|\cos\theta$$

$$\rightarrow \cancel{|\vec{X}|^2} - \vec{X}\cdot\vec{Y} - \vec{X}\cdot\vec{Y} + \cancel{|\vec{Y}|^2} = \cancel{|\vec{X}|^2} + \cancel{|\vec{Y}|^2} - 2|\vec{X}||\vec{Y}|\cos\theta$$

$$\rightarrow -2(\vec{X}\cdot\vec{Y}) = -2|\vec{X}||\vec{Y}|\cos\theta$$

$$\rightarrow \boxed{\vec{X}\cdot\vec{Y} = |\vec{X}||\vec{Y}|\cos\theta}$$

or

$$\boxed{\cos\theta = \frac{\vec{X}\cdot\vec{Y}}{|\vec{X}||\vec{Y}|}}$$