Math 21C
Kouba
Discussion Sheet 10

1.) Find and classify critical points as determining relative maximums, relative minimums, or saddle points.

a.) $z = 3x^2 - 6xy + y^2 + 12x - 16y + 1$
b.) $z = x^2y - x^2 - 2y^2$
c.) $z = x^2 - 8\ln(xy) + y^2$
d.) $z = 3x^2y - 6x^2 + y^3 - 6y^2$

2.) Find the point on the plane $x + 2y + 3z = 6$ nearest the origin.

3.) Determine the dimensions and minimum surface area of a closed rectangular box with volume 8 ft.^{3}

4.) Determine the dimensions and minimum surface area of the closed right circular cylinder with volume 16 π ft.^{3}

5.) Material for the top and bottom of a rectangular box costs \$4/ft.^{2} and that for the sides costs \$2/ft.^{2} Determine the dimensions of the least expensive box of volume 16 ft.^{3}

6.) Among all open (no top) rectangular boxes with surface area 300 in.^{2}, determine the dimensions of the box of maximum volume.

7.) Determine the absolute extrema for each function on the indicated region.

a.) $f(x, y) = 2x + 4y + 12$ on

i.) the triangle with vertices $(0, 0), (0, 3)$, and $(3, 0)$ and its interior.
ii.) the circle $x^2 + y^2 = 4$ and its interior.

b.) $f(x, y) = xy - x - 3y$ on the triangle with vertices $(0, 0), (0, 4)$, and $(5, 0)$ and its interior.

c.) $f(x, y) = x^2 - 3y^2 - 2x + 6y$ on the square with vertices $(0, 0), (0, 2), (2, 0)$ and $(2, 2)$ and its interior.

8.) Use Lagrange multipliers to determine the extreme values for each of the following.

a.) Minimize $f(x, y) = x^2 + y^2$ subject to $2x + 4y = 5$.

b.) Maximize $f(x, y) = x^2 - y^2$ subject to $y = x^2$.

c.) Maximize and minimize $f(x, y) = 3x + 4y + 2$ subject to $x^2 + y^2 = 9$.

d.) Minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x + 2z = 4$ and $x + y = 8$.

"Do just once what others say you can't do, and you will never pay attention to their limitations again." – James R. Cook