Section 12.5

1.) point $P = (3,-4,-1)$, vector $\vec{A} = \overrightarrow{(1,1,1)}$, so

line $L: \begin{cases} x = 3 + (1)t = 3 + t \\ y = -4 + (1)t = -4 + t \\ z = -1 + (1)t = -1 + t \end{cases}$

4.) points $P = (1,2,0)$, $Q = (1,1,-1)$ and vector $\overrightarrow{PQ} = \overrightarrow{(0,-1,-1)}$, so line

$L: \begin{cases} x = 1 + (0)t = 1 \\ y = 2 + (-1)t = 2 - t \\ z = 0 + (-1)t = -t \end{cases}$

6.) point $P = (3,-2,1)$ and $\parallel$ to line

$L: \begin{cases} x = 1 + 2t \\ y = 2 - t \\ z = 3t \end{cases}$ so $\parallel$ vector is $\vec{A} = \overrightarrow{(2,-1,3)}$

and line is

$M: \begin{cases} x = 3 + (2)t = 3 + 2t \\ y = -2 + (-1)t = -2 - t \\ z = 1 + (3)t = 1 + 3t \end{cases}$

7.) point $P = (1,1,1)$ and $\parallel$ to z-axis so $\parallel$ vector is $\vec{A} = \overrightarrow{(0,0,1)}$, and line is

$L: \begin{cases} x = 1 + (0)t = 1 \\ y = 1 + (0)t = 1 \\ z = 1 + (1)t = 1 + t \end{cases}$

8.) point $P = (2,4,5)$ and $\perp$ to plane

$3X + 7Y - 5Z = 21$; plane has $\perp$ vector $\vec{A} = \overrightarrow{(3,7,-5)}$, so line is

$$L: \begin{cases} x = 2 + (3)t = 2 + 3t \\ y = 4 + (7)t = 4 + 7t \\ z = 5 + (-5)t = 5 - 5t \end{cases}$$

10.) $\vec{u} = \overrightarrow{(1,2,3)}$, $\vec{v} = \overrightarrow{(3,4,5)}$ so

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = (10-12)\vec{i} - (5-9)\vec{j} + (4-6)\vec{k}$$

$= -2\vec{i} + 4\vec{j} - 2\vec{k}$ is $\perp$ to $\vec{u}$ and $\vec{v}$, and point $P = (2,3,0)$ so line

$$L: \begin{cases} X = 2 + (-2)t = 2 - 2t \\ Y = 3 + (4)t = 3 + 4t \\ Z = 0 + (-2)t = -2t \end{cases}$$

21.) point $P = (0,2,-1)$ and $\perp$ vector $\vec{u} = \overrightarrow{(3,-2,-1)}$, so plane is

$3(x-0) - 2(Y-2) - 1(Z-(-1)) = 0 \longrightarrow$

$3X - 2Y + 4 - Z - 1 = 0 \longrightarrow$

$3X - 2Y - Z = -3$.

22.) plane $3X + Y + Z = 7$ has $\perp$ vector $\vec{u} = (3,1,1)$, and point $P = (1,-1,3)$, so new plane is

$3(x-1) + 1\cdot(Y-(-1)) + 1\cdot(Z-3) = 0 \longrightarrow$

$3x - 3 + y + 1 + z - 3 = 0 \rightarrow$

$3x + y + z = 5$

24.) points $P = (2,4,5)$, $Q = (1,5,7)$, $R = (-1,6,8)$ so vectors $\vec{PQ} = \overrightarrow{(-1,1,2)}$ and $\vec{PR} = \overrightarrow{(-3,2,3)}$, so $\perp$ vector to plane is

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix} = (3-4)\vec{i} - (-3+6)\vec{j} + (-2+3)\vec{k}$$

$= -\vec{i} - 3\vec{j} + \vec{k}$; equation of plane is $-1(x-2) - 3(y-4) + 1(z-5) = 0 \rightarrow$

$-x + 2 - 3y + 12 + z - 5 = 0 \rightarrow$

$-x - 3y + z = -9$

25.) Line $L: \begin{cases} x = 5 + t \\ y = 1 + 3t \\ z = 4t \end{cases}$ has $\parallel$ vector

$\vec{A} = \overrightarrow{(1,3,4)}$, so plane has $\perp$ vector $\vec{A} = \overrightarrow{(1,3,4)}$, so plane through point $P = (2,4,5)$ is

$1 \cdot (x-2) + 3 \cdot (y-4) + 4 \cdot (z-5) = 0 \rightarrow$

$x - 2 + 3y - 12 + 4z - 20 = 0 \rightarrow$

$x + 3y + 4z = 34$

28.) Lines
$$L_1: \begin{cases} x = t \\ y = 2 - t \\ z = 1 + t \end{cases} \quad \text{and} \quad L_2: \begin{cases} x = 2 + 2s \\ y = 3 + s \\ z = 6 + 5s \end{cases} \; ;$$

if lines intersect, then

$$\left.\begin{aligned} t &= 2 + 2s \\ 2 - t &= 3 + s \end{aligned}\right\} (\text{add}) \rightarrow 2 = 5 + 3s \rightarrow$$

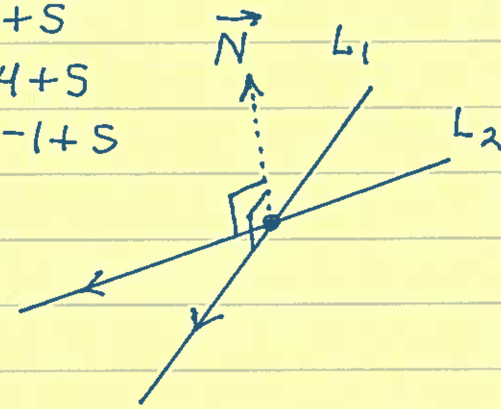
$s = -1$ and $t = 0$, so pt. of $\cap$ is $\boxed{P = (x, y, z) = (0, 2, 1)}$; vector $\parallel$ to L_1 is $\vec{A} = \overrightarrow{(1, -1, 1)}$, vector $\parallel$ to L_2 is $\vec{B} = \overrightarrow{(2, 1, 5)}$, so vector $\perp$ to plane is

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & 5 \end{vmatrix} = (-5-1)\vec{i} - (5-2)\vec{j} + (1+2)\vec{k}$$

$= \underline{\underline{-6\vec{i} - 3\vec{j} + 3\vec{k}}}$; then equation of plane is

$$-6 \cdot (x - 0) - 3(y - 2) + 3(z - 1) = 0 \rightarrow$$
$$-6x - 3y + 6 + 3z - 3 = 0 \rightarrow$$
$$-6x - 3y + 3z = -3 \rightarrow$$
$$2x + y - z = 1 \; .$$

30.) $L_1: \begin{cases} x = t \\ y = 3-3t \\ z = -2-t \end{cases}$ $L_2: \begin{cases} x = 1+s \\ y = 4+s \\ z = -1+s \end{cases}$



Find pt. of intersection:

$\begin{cases} t = 1+s \\ 3-3t = 4+s \end{cases}$

$3 - 3(1+s) = 4+s$

$\rightarrow 3-3-3s = 4+s \rightarrow 4 = -4s \rightarrow s = -1, t = 0$

so pt. of intersection is $\boxed{(0,3,-2)}$; direction vectors for lines L_1 and L_2 are $\vec{v} = \overrightarrow{(1,-3,-1)}$ and $\vec{w} = \overrightarrow{(1,1,1)}$, so normal vector to plane is

$$\vec{N} = \vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & -1 \\ 1 & 1 & 1 \end{vmatrix} = (-3+1)\vec{i} - (1+1)\vec{j} + (1+3)\vec{k}$$

$$= -2\vec{i} - 2\vec{j} + 4\vec{k},$$

i.e., $\boxed{\vec{N} = \overrightarrow{(-2,-2,4)}}$; so equation of plane is $-2(x-0) - 2(y-3) + 4(z+2) = 0$

$\rightarrow -2x - 2y + 6 + 4z + 8 = 0$

$\rightarrow -2x - 2y + 4z = -14$.

31.) plane $2x + y - z = 3$ has $\perp$ vector $\vec{A} = \overrightarrow{(2,1,-1)}$; plane $x + 2y + z = 2$ has $\perp$ vector $\vec{B} = \overrightarrow{(1,2,1)}$, so plane $\parallel$ to the line of intersection of these planes is

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (1+2)\,\vec{i} - (2+1)\,\vec{j} + (4-1)\,\vec{k}$$

$= \underline{\underline{3\,\vec{i} - 3\,\vec{j} + 3\,\vec{k}}}$; so plane $\perp$ to $\vec{A} \times \vec{B}$ and through point $P = (2,1,-1)$ is

$3(x-2) - 3(y-1) + 3(z+1) = 0 \rightarrow$

$(x-2) - (y-1) + (z+1) = 0 \rightarrow$

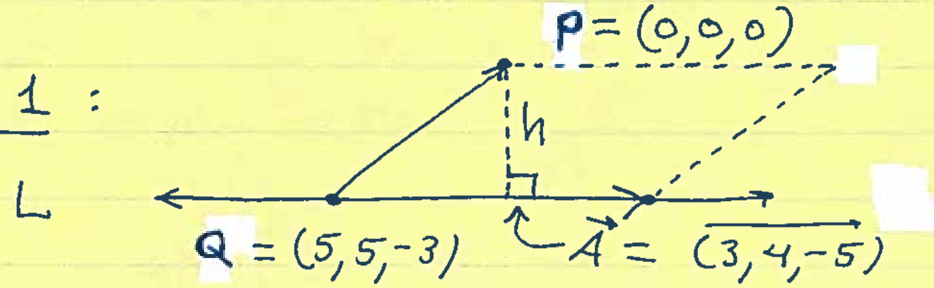
$x - y + z = 0$

34.) point $P = (0,0,0)$, line

$$L: \begin{cases} x = 5 + 3t \\ y = 5 + 4t \\ z = -3 - 5t \end{cases}$$

; distance from point P to line L is :

<u>Method 1</u> :



Point $Q = (5,5,-3)$ is on line L and vector $\vec{A} = \overrightarrow{(3,4,-5)}$ is $\parallel$ to L ; vector $\overrightarrow{QP} = \overrightarrow{(-5,-5,3)}$; area of parallelogram formed by $\overrightarrow{QP}$ and $\vec{A}$ is

$|\overrightarrow{QP} \times \vec{A}| = (\text{base})(\text{height})$

$= |\vec{A}| \cdot h \Rightarrow$

$$h = \frac{|\overrightarrow{QP} \times \vec{A}|}{|\vec{A}|} \quad ; \quad \overrightarrow{QP} \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -5 & 3 \\ 3 & 4 & -5 \end{vmatrix}$$

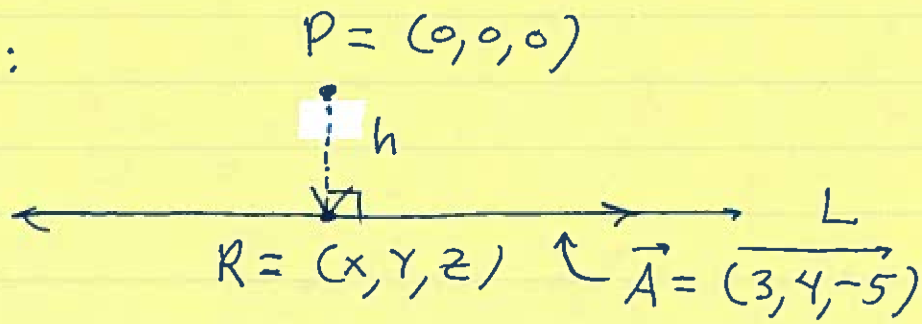
$$= (25-12)\vec{i} - (25-9)\vec{j} + (-20+15)\vec{k}$$

$$= 13\vec{i} - 16\vec{j} - 5\vec{k}, \text{ so}$$

$$|\overrightarrow{QP} \times \vec{A}| = \sqrt{169+256+25} = \sqrt{450} = 15\sqrt{2};$$

then $h = \dfrac{|\overrightarrow{QP} \times \vec{A}|}{|\vec{A}|} = \dfrac{15\sqrt{2}}{\sqrt{50}} = \dfrac{15\sqrt{2}}{5\sqrt{2}} = 3$

<u>Method 2</u>:



Find point $R = (x,y,z)$ on line L so that vector $\overrightarrow{PR} = \overrightarrow{(x,y,z)}$ is $\perp$ to $\vec{A}$ $\Rightarrow$ $\overrightarrow{PR} \cdot \vec{A} = 0 \Rightarrow$

$$\overrightarrow{(x,y,z)} \cdot \overrightarrow{(3,4,-5)} = 0 \Rightarrow$$

$$3x + 4y - 5z = 0 \Rightarrow$$

$$3(5+3t) + 4(5+4t) - 5(-3-5t) = 0 \Rightarrow$$

$$15 + 9t + 20 + 16t + 15 + 25t = 0 \Rightarrow$$

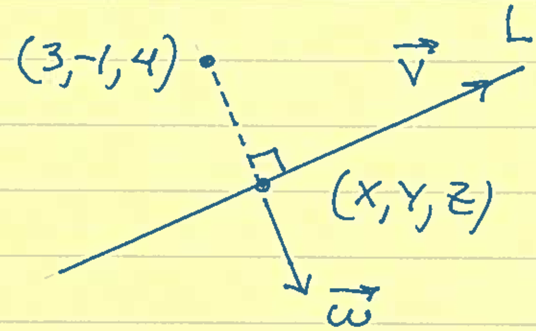
$$50t + 50 = 0 \longrightarrow \underline{\underline{t=-1}}; \text{ then}$$

point $R = (2,1,2)$ and distance from point P to R is

$$h = \sqrt{(2-0)^2 + (1-0)^2 + (2-0)^2} = \sqrt{9} = 3$$

37.) pt. $(3,-1,4)$, line L: $\begin{cases} x = 4-t \\ y = 3+2t \\ z = -5+3t \end{cases}$

direction vector for line is



$\vec{V} = \overrightarrow{(-1,2,3)}$; vector through points $(3,-1,4)$ and (x,y,z) is

$\vec{w} = \overrightarrow{(x-3, y+1, z-4)}$

$= \overrightarrow{((4-t)-3, (3+2t)+1, (-5+3t)-4)}$

$= \overrightarrow{(1-t, 4+2t, 3t-9)}$; $\vec{V} \perp \vec{w}$ so

$\vec{V} \cdot \vec{w} = 0 \rightarrow (-1)(1-t)+(2)(4+2t)+(3)(3t-9)=0$

$\rightarrow -1+t+8+4t+9t-27=0$

$\rightarrow 14t = 20 \rightarrow t = \frac{20}{14} = \frac{10}{7}$ so point

$(x,y,z) = (4-\frac{10}{7}, 3+\frac{20}{7}, -5+\frac{30}{7})$

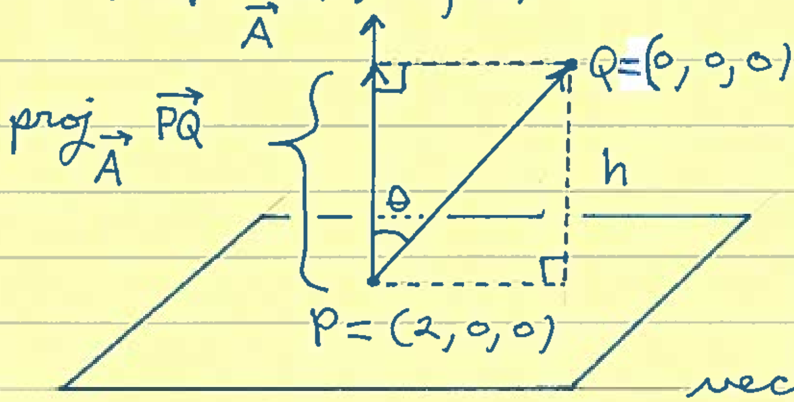
$= (\frac{18}{7}, \frac{41}{7}, -\frac{5}{7})$; find distance between points $(3,-1,4)$ and $(\frac{18}{7}, \frac{41}{7}, -\frac{5}{7})$:

$$\text{Dist.} = \sqrt{(3-\tfrac{18}{7})^2 + (-1-\tfrac{41}{7})^2 + (4- -\tfrac{5}{7})^2}$$

$$= \sqrt{(\tfrac{3}{7})^2 + (-\tfrac{48}{7})^2 + (\tfrac{33}{7})^2}$$

$$= \sqrt{\frac{3402}{49}} = \frac{\sqrt{3402}}{7}.$$

40.) pt. $(0,0,0)$, plane : $3X+2Y+6Z=6$



plane $3X+2Y+6Z=6$ contains the point $P=(2,0,0)$; vector $\vec{A}=\overrightarrow{(3,2,6)}$ is $\perp$ to plane; then distance from Q to plane is

$$h = \left|\text{proj}_{\vec{A}}\,\overrightarrow{PQ}\right|$$

$$= |\overrightarrow{PQ}|\cdot|\cos\theta|$$

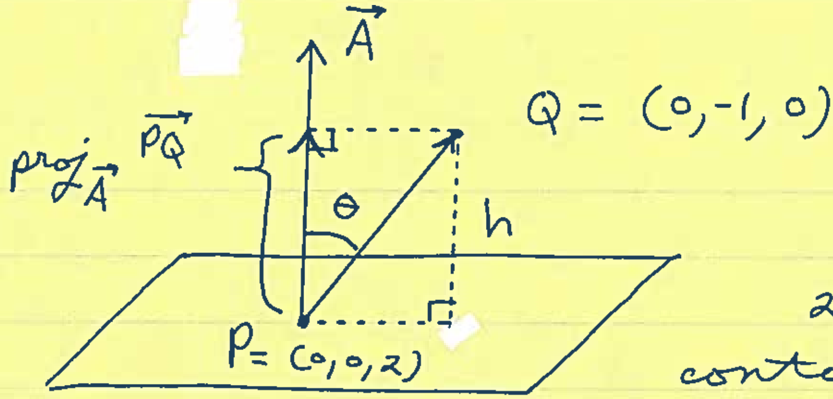
$$= \cancel{|\overrightarrow{PQ}|}\cdot\frac{|\overrightarrow{PQ}\cdot\vec{A}|}{\cancel{|\overrightarrow{PQ}|}\,|\vec{A}|}$$

$$= \frac{|\overrightarrow{(-2,0,0)}\cdot\overrightarrow{(3,2,6)}|}{\sqrt{3^2+2^2+6^2}}$$

$$= \frac{|-6|}{\sqrt{49}}$$

$$= \frac{6}{7}.$$

43.)



plane $2X + Y + 2Z = 4$ contains the point $P = (0,0,2)$; vector $\vec{A} = \overrightarrow{(2,1,2)}$ is $\perp$ to plane; then distance from Q to plane is

$$h = |proj_{\vec{A}} \vec{PQ}|$$

$$= |\vec{PQ}| \cdot |\cos\theta|$$

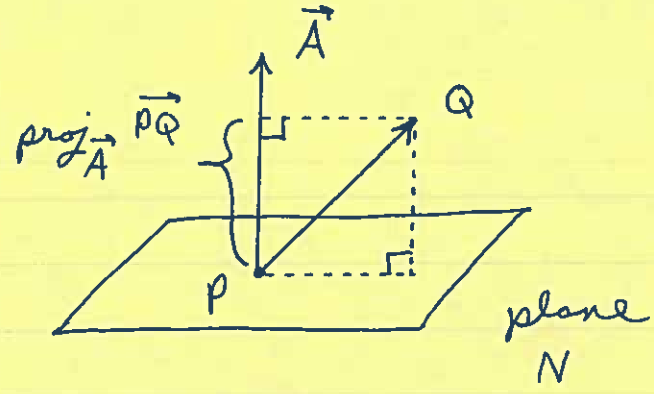
$$= \cancel{|\vec{PQ}|} \cdot \frac{|\vec{PQ} \cdot \vec{A}|}{\cancel{|\vec{PQ}|}\,|\vec{A}|}$$

$$= \frac{|\overrightarrow{(0,-1,-2)} \cdot \overrightarrow{(2,1,2)}|}{|\overrightarrow{(2,1,2)}|}$$

$$= \frac{|0-1-4|}{\sqrt{2^2+1^2+2^2}} = \frac{5}{3}$$

45.) Planes $M: X + 2Y + 6Z = 1$ and $N: X + 2Y + 6Z = 10$ are $\parallel$; point $Q = (1,0,0)$ is on M and point $P = (10,0,0)$ is on N; vector $\vec{A} = \overrightarrow{(1,2,6)}$ is $\perp$ to N; now find distance from point Q to plane N; then distance

$$h = |proj_{\vec{A}} \vec{PQ}|$$

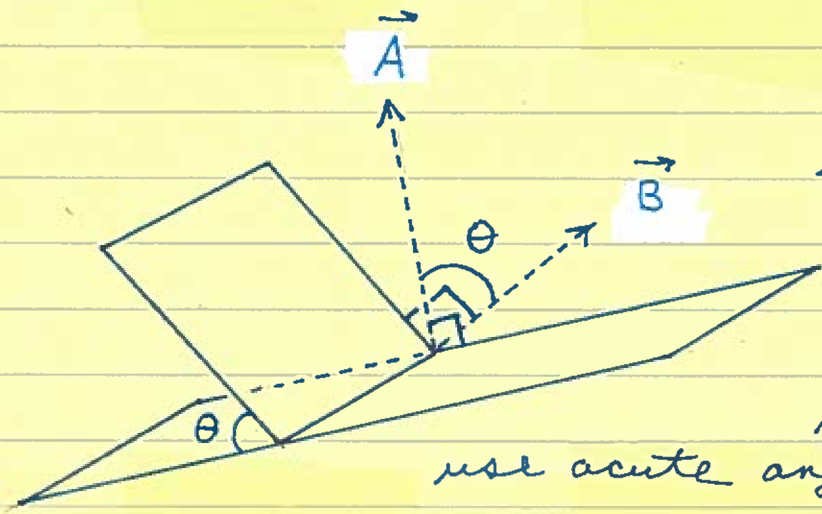


$$= |\vec{PQ}||\cos\theta|$$

$$= \cancel{|\vec{PQ}|} \cdot \frac{|\vec{PQ} \cdot \vec{A}|}{\cancel{|\vec{PQ}|}|\vec{A}|}$$

$$= \frac{|\overrightarrow{(-9,0,0)} \cdot \overrightarrow{(1,2,6)}|}{|\overrightarrow{(1,2,6)}|} = \frac{|-9|}{\sqrt{41}} = \frac{9}{\sqrt{41}}$$

47.)



The angle between planes is the angle between their normal vectors; use acute angle, $0° \le \theta \le 90°$.

Plane $X + Y = 1$ has $\perp$ vector $\vec{A} = \overrightarrow{(1,1,0)}$; plane $2X + Y - 2Z = 2$ has $\perp$ vector $\vec{B} = \overrightarrow{(2,1,-2)}$; the angle between the planes is the angle (acute : $0° \le \theta \le 90°$) determined by the $\perp$ vectors :

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{2+1-0}{\sqrt{2} \cdot \sqrt{9}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$\theta = 45°$

51.) Plane $2X+2Y-Z=3$ has $\perp$ vector $\vec{A} = \overrightarrow{\langle 2,2,-1\rangle}$; plane $X+2Y+Z=2$ has $\perp$ vector $\vec{B} = \overrightarrow{\langle 1,2,1\rangle}$. The angle between the planes is the angle determined by their $\perp$ vectors :

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{2+4-1}{\sqrt{9}\cdot\sqrt{6}} = \frac{5}{3\sqrt{6}}, \text{ so}$$

$$\theta = \arccos\left(\frac{5}{3\sqrt{6}}\right) \approx 47.12^\circ$$

55.) plane $X+Y+Z=2$, line $L: \begin{cases} x = 1+2t \\ y = 1+5t \\ z = 3t \end{cases}$;

the point of $\cap$ is given by

$X+Y+Z = (1+2t)+(1+5t)+(3t) = 2 \Rightarrow$

$10t = 0 \longrightarrow t = 0 \longrightarrow$ point is

$(x,y,z) = (1,1,0)$

59.) Plane $x - 2y + 4z = 2$ has $\perp$ vector $\vec{A} = \overrightarrow{(1,-2,4)}$; plane $x + y - 2z = 5$ has $\perp$ vector $\vec{B} = \overrightarrow{(1,1,-2)}$; then the line forming the $\cap$ of these planes is $\parallel$ to the vector

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 4 \\ 1 & 1 & -2 \end{vmatrix} = (4-4)\vec{i} - (-2-4)\vec{j} + (1+2)\vec{k}$$

$$= \underline{\underline{6\vec{j} + 3\vec{k}}}$$; now find a point on this line :

$$\left.\begin{aligned} x - 2y + 4z &= 2 \\ x + y - 2z &= 5 \end{aligned}\right\} \quad \left.\begin{aligned} \underline{x = 2 + 2y - 4z} \\ x = 5 - y + 2z \end{aligned}\right\}$$

$2 + 2y - 4z = 5 - y + 2z \longrightarrow$

$3y = 3 + 6z \longrightarrow \underline{y = 1 + 2z}$; now let z be ANY number : $z = 0 \rightarrow$

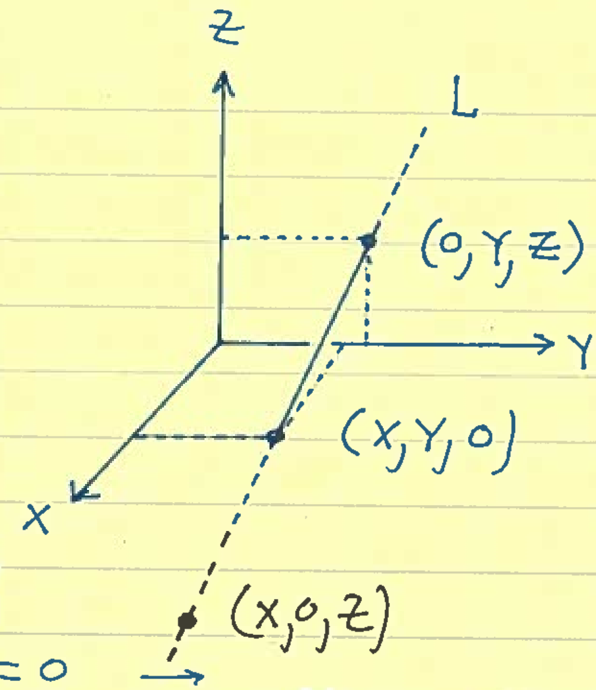
$y = 1 \longrightarrow x = 4$, so point

$(x,y,z) = \underline{\underline{(4,1,0)}}$ lies on BOTH planes ;

now the line of intersection is given by

$$L: \begin{cases} x = 4 + (0)t = 4 \\ y = 1 + (6)t = 1 + 6t \\ z = 0 + (3)t = 3t \end{cases}$$

65.) line L : $\begin{cases} X = 1+2t \\ Y = -1-t \\ Z = 3t \end{cases}$



meet XY-plane : $Z=0$

$\rightarrow Z = 3t = 0 \rightarrow t = 0$

$\rightarrow X = 1+2(0) = 1$

and $Y = -1-(0) = -1$

so point is $\boxed{(1,-1,0)}$

meet YZ-plane : $X = 0 \longrightarrow$

$X = 1+2t = 0 \rightarrow t = -1/2 \longrightarrow Y = -1-\left(\frac{-1}{2}\right) = -1/2$

and $Z = 3t = 3(-1/2) = -3/2$ so point is $\boxed{(0, -1/2, -3/2)}$.

meet XZ-plane : $Y = 0 \longrightarrow$

$Y = -1-t = 0 \longrightarrow t = -1 \longrightarrow X = 1+2t = 1+2(-1) = -1$

and $Z = 3t = 3(-1) = -3$ so point is

$\boxed{(-1, 0, -3)}$.

67.) line L = $\begin{cases} X = 1-2t \\ Y = 2+5t \\ Z = -3t \end{cases}$ has direction vector

$\vec{A} = \overrightarrow{(-2, 5, -3)}$; plane $2X+Y-Z=8$ has $\perp$

vector $\vec{B} = \overrightarrow{(2,1,-1)}$; then line and plane

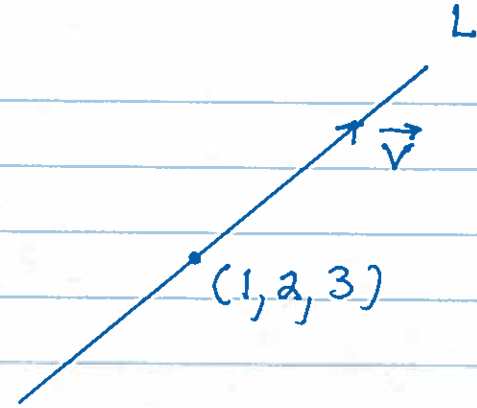
are parallel if and only if $\vec{A} \perp \vec{B}$:

$\vec{A} \cdot \vec{B} = -4+5+3 = 4 \neq 0$, so

$\vec{A}$ not $\perp$ $\vec{B}$ and line is not parallel

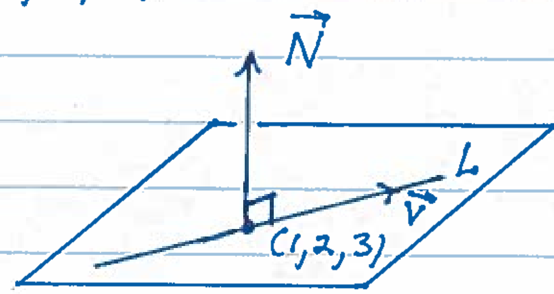
to plane .

69.) line $L = \begin{cases} X = 1+t \\ Y = 2-t \\ Z = 3+2t \end{cases}$



The point (1, 2, 3) lies on line L (let $t=0$); vector $\vec{V} = \overrightarrow{(1,-1,2)}$ is a direction vector for line L; find plane containing point (1, 2, 3) and line L :

i.) Find vector $\vec{N} \perp \vec{V}$:



Let $\vec{N} = \overrightarrow{(a,b,c)}$ then

$\vec{N} \cdot \vec{V} = 0 \rightarrow$

$(a)(1)+(b)(2)+(c)(3) = 0$

$\rightarrow a+2b+3c = 0$, then $a=1$, $b=1$, and $c=-1$, so $\vec{N} = \overrightarrow{(1,1,-1)}$ and plane is

$$(1)(X-1)+(1)(Y-2)+(-1)(Z-3) = 0 \rightarrow$$

$$X-1+Y-2-Z+3 = 0 \rightarrow$$

$$\boxed{X+Y-Z=0}$$

ii.) Find another vector $\vec{N} \perp \vec{V}$:

$\vec{N} \cdot \vec{V} = 0 \rightarrow a+2b+3c=0$, then $a=2$, $b=-1$, and $c=0$, so $\vec{N} = \overrightarrow{(2,-1,0)}$ and plane is

$$(2)(X-1)+(-1)(Y-2)+(0)(Z-3) = 0 \rightarrow$$

$$2X-2-Y+2 = 0 \rightarrow$$

$$\boxed{2X-Y=0}$$. These planes intersect in given line.