Section 14.1

1.) d.) \( f(x, y) = x^2 + xy^3 \rightarrow f(3, -2) = (3)^2 + 3(-2)^3 = 9 - 24 = -15 \)

4.) c.) \( f(x, y, z) = \sqrt{49-x^2-y^2-z^2} \rightarrow \)
\( f(-1, 2, 3) = \sqrt{49-(-1)^2-(2)^2-(3)^2} \)
\( = \sqrt{49-1-4-9} = \sqrt{35} \)

5.) \( f(x, y) = \sqrt{y-x-2} ; \) need \( y-x-2 \geq 0 \)

\( \rightarrow y \geq x + 2 \Rightarrow \)

Domain: all pts. \( (x, y) \) on or above the line \( y = x + 2 \)

7.) \( f(x, y) = \frac{(x-1)(y+2)}{(y-x)(y-x^3)} \rightarrow \)
\( y \neq x, y \neq x^3 \;

Domain: all pts. \( (x, y) \) NOT on graphs \( y = x, y = x^3 \)
10. \( f(x, y) = \ln \left( x + y + x - y - 1 \right) \)
   \[= \ln \left( (x-1)(y+1) \right) \]
   need \( (x-1)(y+1) > 0 \) 
   \( (x-1)(y+1) = 0 \) when \( x = 1 \) or \( y = -1 \).
   now choose test point in each of the 4 regions.
   Domain: all pts. \((x, y)\) in the shaded regions.

11. \( f(x, y) = \sqrt{(x^2-4)(y^2-9)} = \sqrt{(x-2)(x+2)(y-3)(y+3)} \)
   need \( (x-2)(x+2)(y-3)(y+3) \geq 0 \) 
   \( (x-2)(x+2)(y-3)(y+3) = 0 \) when \( x = 2, x = -2, y = 3, \) or \( y = -3 \).
   now choose test point in each of the 9 regions.
   Domain: all pts. \((x, y)\) on the lines \( x = 2, x = -2, y = 3, y = -3 \) or in shaded regions.
18.) \( f(x, y) = \sqrt{y-x} \)
   a.) **Domain:** \( y-x \geq 0 \)
      \( \Rightarrow \) all pts. \((x, y)\) with \( y \geq x \)

b.) Consider all pts. \((0, y)\) for \( 0 \leq y < \infty \); for these pts., the \( z \)-value is \( z = \sqrt{y} \) and \( 0 \leq z < \infty \). It follows (since \( \sqrt{x-y} \geq 0 \)) that the **Range** of \( f \) is \( 0 \leq z < \infty \).

19.) \( f(x, y) = 4(x^2 + 9y^2) \)
   a.) **Domain:**
      all pts. \((x, y)\)
   b.) Consider all pts. \((x, 0)\) for \(-\infty < x < \infty \); for these pts., the \( z \)-value is \( z = 4x^2 \) and \( 0 \leq z < \infty \). It follows (since \( 4x^2 + 9y^2 \geq 0 \)) that the **Range** of \( f \) is \( 0 \leq z < \infty \).

23.) \( f(x, y) = \frac{1}{\sqrt{16-x^2-y^2}} \)
   a.) **Domain:** \( 16-x^2-y^2 > 0 \)
      \( \Rightarrow \) \( x^2 + y^2 < 16 \) so domain is set of pts. \((x, y)\) inside the circle \( x^2 + y^2 = 4^2 \)
b.) Consider all pts. (x, y), where -4 < x < 4; for these pts. the z-value is \( z = \frac{1}{\sqrt{16 - x^2}} \); note that \( z = \frac{1}{4} \) if \( x = 0 \) and \( \lim_{x \to 4^-} z = \lim_{x \to 4^-} \frac{1}{\sqrt{16 - x^2}} = \frac{1}{10} = +\infty \). The z-values range for \( z = \frac{1}{4} \) to \( +\infty \); since \( \frac{1}{4} \leq \frac{1}{\sqrt{16 - x^2}} \), it follows that the Range of \( f \) is \( \frac{1}{4} \leq z < \infty \).

24.) \( f(x, y) = \sqrt{9 - x^2 - y^2} \)

a.) **Domain:** \( 9 - x^2 - y^2 \geq 0 \)

\[ x^2 + y^2 \leq 9 \]

so domain is set of all pts. \((x, y)\) on or inside the circle \( x^2 + y^2 = 3^2 \).

b.) Consider that \( z = \sqrt{9 - x^2 - y^2} \) \( \Rightarrow z^2 = 9 - x^2 - y^2 \) \( \Rightarrow x^2 + y^2 + z^2 = 3^2 \) is a sphere of radius 3 centered at \((0, 0, 0)\); the \( z = \sqrt{9 - x^2 - y^2} \) is the top half of the sphere, so the **Range** of \( f \) is \( 0 \leq z \leq 3 \).

25.) \( f(x, y) = \ln(x^2 + y^2) \)

a.) **Domain:** \( x^2 + y^2 > 0 \) so domain is set of all pts. \((x, y)\) except \((0, 0)\);
b.) Consider all pts. $(x, 0)$ where $0 < x < \infty$; for these pts. the $z$-value is $z = \ln x^2 \rightarrow z = 2 \ln x$; these $z$-values range from $-\infty$ to $+\infty$; it follows that the range of $f$ is $-\infty < z < \infty$.

\[ f(x, y) = \ln (4 - x^2 - y^2) \]

a.) Domain: $4 - x^2 - y^2 > 0 \rightarrow x^2 + y^2 < 4$ so domain is set of all pts. $(x, y)$ inside the circle $x^2 + y^2 = 4$.

b.) Range: Consider $x^2$-trace of surface $z = \ln (4 - x^2 - y^2) \rightarrow y = 0 \rightarrow $
\[ z = \ln (4 - x^2) \]

If \( z = 0 \):
\[
0 = \ln (4 - x^2) \\
\rightarrow 4 - x^2 = 1 \\
\rightarrow x^2 = 3 \\
\rightarrow x = \pm \sqrt{3}
\]

If \( x = 0 \):
\[ z = \ln 4 \]

so **range** is all values of \( z \) satisfying \(-\infty < z \leq \ln 4\)