Math 21C
Kouba
Why Does Right Hand Rule for $\vec{A} \times \vec{B}$ Work?

Justification: Let $\vec{A} = (a_1, a_2, a_3)$ and $\vec{B} = (b_1, b_2, b_3)$ and let $0 < \theta < 180^\circ$ be the angle between them.

"Translate" vectors $\vec{A}$ and $\vec{B}$ to the xy-plane in such a way that:

i) their tails are at the origin,
ii) $\vec{A}$ points in the direction of $\hat{i}$,
iii) their lengths are preserved,
and iv) the angle between them remains $\theta$.

Call the translated vectors $\vec{A}^*$ and $\vec{B}^*$.

\[ \begin{align*}
\vec{A}^* &= (a_1, 0, 0) \\
\vec{B}^* &= (b \cos \theta, b \sin \theta, 0)
\end{align*} \]

It follows that $\vec{A}^* = (a, 0, 0)$ and $\vec{B}^* = (b \cos \theta, b \sin \theta, 0)$.

\[ a = |\vec{A}^*| = |\vec{A}| > 0 \quad \text{and} \quad b = |\vec{B}^*| = |\vec{B}| > 0. \]

Then

\[ \vec{A}^* \times \vec{B}^* = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
0 & a & 0 \\
0 & 0 & b \cos \theta \sin \theta
\end{vmatrix} = (ab \sin \theta) \vec{k} \]

where $ab \sin \theta > 0$. This vector points in the direction of the positive z-axis, consistent with the Right Hand Rule.