Math 21D
Kouba
Applications of Triple Integrals

Let R be a solid region in three-dimensional space and let $\delta(P)$ be the density of the region at point $P = (x, y, z)$.

1.) VOLUME : $\int_R 1\, dV$ represents the *volume* of region R.

2.) AVERAGE VALUE : $\frac{1}{Volume\ of\ R} \int_R f(x, y, z)\, dV$ represents the *average value* of function $w = f(x, y, z)$ over region R.

3.) MASS : $\int_R \delta(P)\, dV$ represents the *mass* of region R.

4.) MOMENT :

a.) $\int_R (x - a)\delta(P)\, dV$ represents the *moment* of region R about the plane $x = a$.

b.) $\int_R (y - b)\delta(P)\, dV$ represents the *moment* of region R about the plane $y = b$.

c.) $\int_R (z - c)\delta(P)\, dV$ represents the *moment* of region R about the plane $z = c$.

5.) CENTER OF MASS, $(\bar{x}, \bar{y}, \bar{z})$:

a.) $\bar{x} = \frac{\int_R x\delta(P)\, dV}{\int_R \delta(P)\, dV}$ represents the *x-coordinate* of the center of mass of region R.

b.) $\bar{y} = \frac{\int_R y\delta(P)\, dV}{\int_R \delta(P)\, dV}$ represents the *y-coordinate* of the center of mass of region R.

c.) $\bar{z} = \frac{\int_R z\delta(P)\, dV}{\int_R \delta(P)\, dV}$ represents the *z-coordinate* of the center of mass of region R.

6.) CENTROID, $(\bar{x}, \bar{y}, \bar{z})$:

a.) $\bar{x} = \frac{\int_R x\, dV}{\int_R 1\, dV}$ represents the *x-coordinate* of the centroid of region R.

b.) $\bar{y} = \frac{\int_R y\, dV}{\int_R 1\, dV}$ represents the *y-coordinate* of the centroid of region R.

c.) $\bar{z} = \dfrac{\int_R z\, dV}{\int_R 1\, dV}$ represents the *z-coordinate* of the center of mass of region R.

NOTE : The formulas for centroid follow immediately from the formulas for center of mass by letting density $\delta(P) = 1$.

7.) MOMENT OF INERTIA : $\displaystyle\int_R (distance)^2 \delta(P)\, dV$ represents the *moment of inertia* of region R, where *distance* refers to the distance from point $P = (x, y, z)$ in region R to either a point or axis (line) of rotation.