

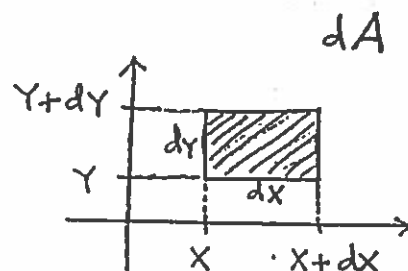
A BRIEF SUMMARY OF MULTIPLE INTEGRATION AND COORDINATE SYSTEMS

Math 21D Kouba

I. Double Integrals

A. Rectangular Coordinates : $dA = dy dx$

B. Polar Coordinates : $dA = (r d\theta) dr = r dr d\theta$

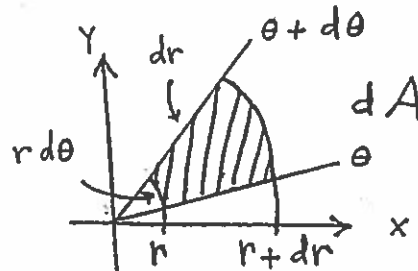


II. Triple Integrals

A. Rectangular Coordinates : $dV = dz dy dx$

B. Cylindrical Coordinates : $dV = dz dr (r d\theta) = r dz dr d\theta$

C. Spherical Coordinates : $dV = d\rho (\rho d\phi) (\rho \sin \phi d\theta) = \rho^2 \sin \phi d\rho d\phi d\theta$



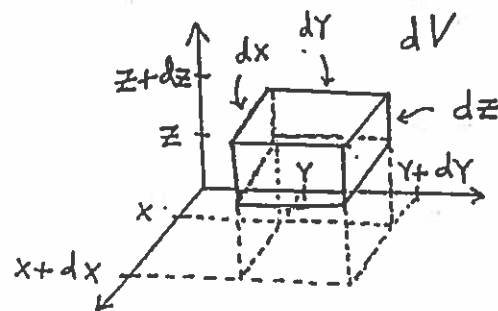
III. Relationships Between Coordinate Systems

A. Two-Dimensional— rectangular (x, y) , polar (r, θ)

$$x = r \cos \theta, y = r \sin \theta$$

and

$$r^2 = x^2 + y^2, \tan \theta = \frac{y}{x}$$



B. Three-Dimensional— rectangular (x, y, z) , cylindrical (r, θ, z) , spherical (ρ, θ, ϕ)

$$r = \rho \sin \phi$$

and

$$x = (\rho \sin \phi) \cos \theta = \rho \cos \theta \sin \phi,$$

$$y = (\rho \sin \phi) \sin \theta = \rho \sin \theta \sin \phi,$$

$$z = \rho \cos \phi$$

and

$$\rho^2 = x^2 + y^2 + z^2$$

