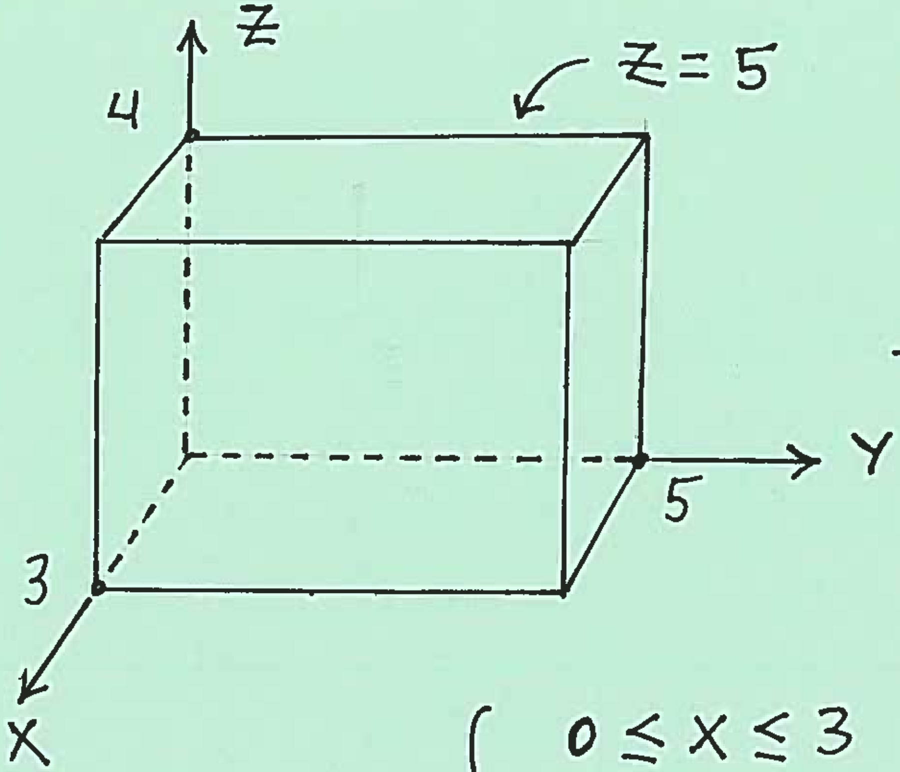
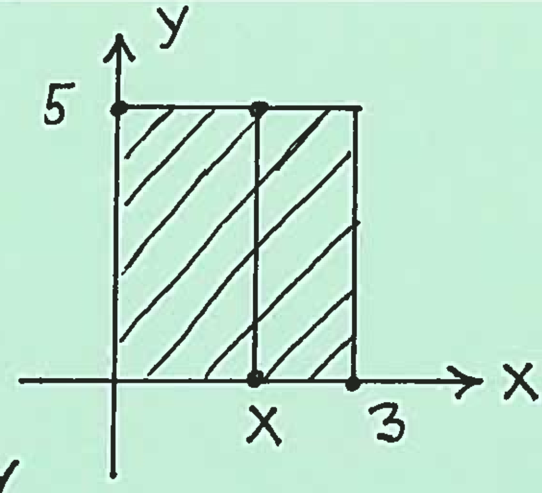
Section 15.4
Thomas Calculus
11th Ed.

Triple Integrals Over Solid Regions R in 3D-Space, Using Rectangular Coordinates

Describing Regions R in 3D-Space

Example: Consider the rectangular box R given in the diagram. Describe R by first projecting R onto the xy-plane.





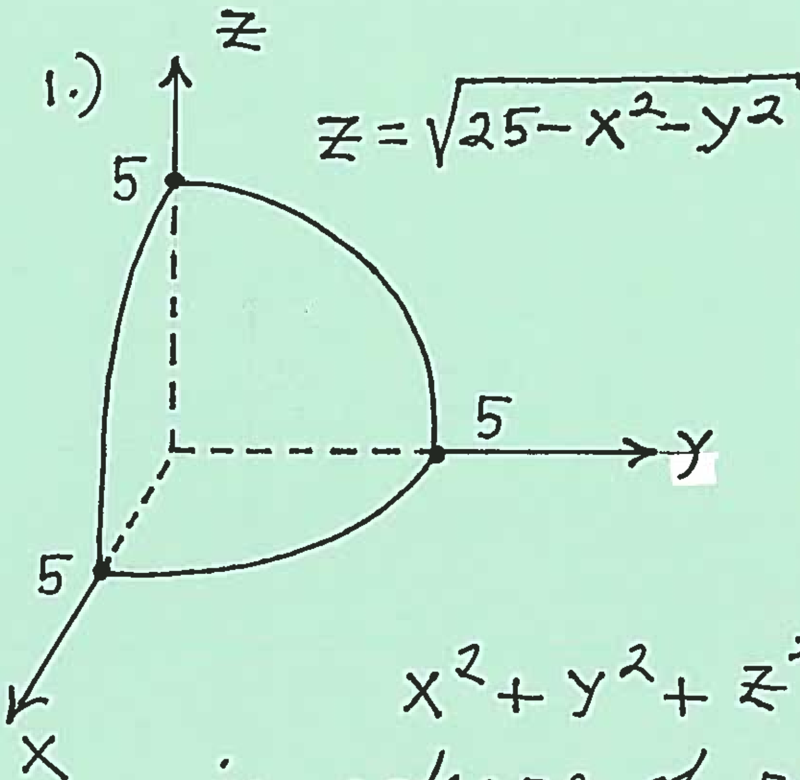
$$R: \begin{cases} 0 \le x \le 3 \\ 0 \le y \le 5 \\ 0 \le z \le 4 \end{cases}$$

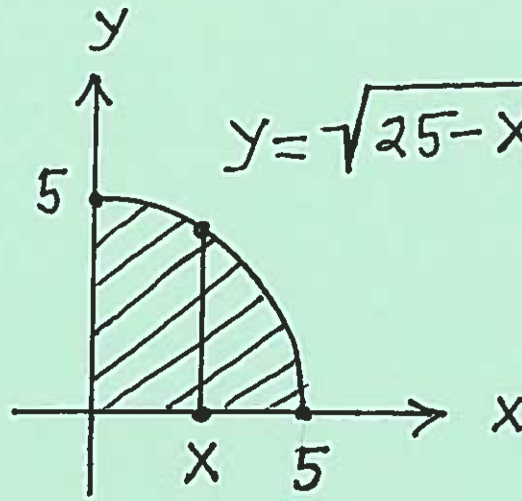
<u>Example</u>: Consider region R in the first octant and inside the sphere $x^2+y^2+z^2=25$

1.) Sketch the solid R.

2.) Describe R by first projecting R onto the xy-plane.

1.)





$$x^2+y^2+z^2 = 25 = 5^2$$

is sphere of radius $r=5$ and center $(0,0,0)$ $\longrightarrow$

$$z=\sqrt{25-x^2-y^2}$$

2.) $R: \begin{cases} 0 \le x \le 5 \\ 0 \le y \le \sqrt{25-x^2} \\ 0 \le z \le \sqrt{25-x^2-y^2} \end{cases}$

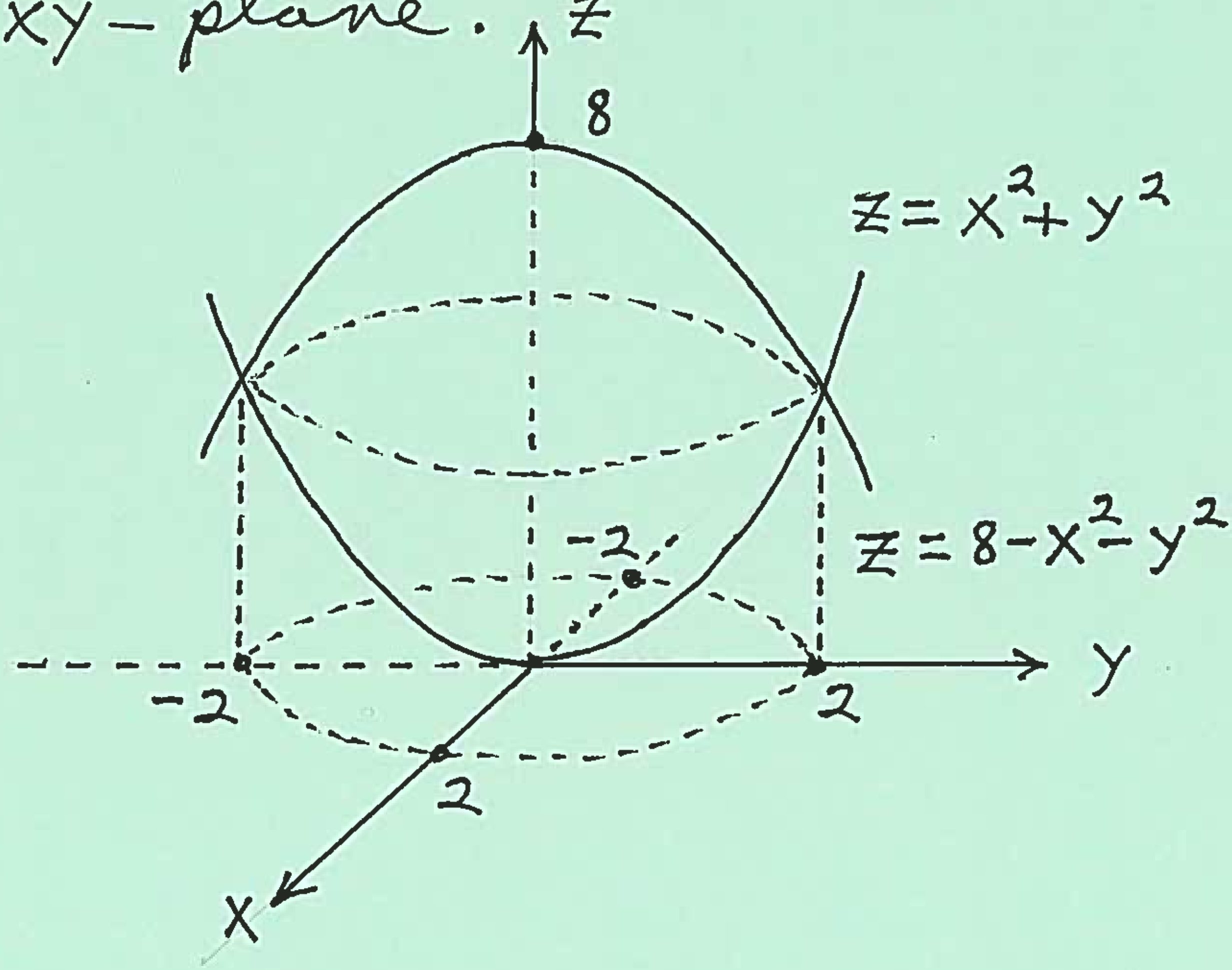
<u>Example</u>: Consider region R bounded by the paraboloids $z=x^2+y^2$ and $z=8-x^2-y^2$.

1.) Sketch the solid R.

2.) Sketch the Projection of R onto the xy-plane.

3.) Describe R by first projecting R onto the xy-plane.

1.)
2.)



3.) $z = x^2 + y^2$ and $z = 8 - x^2 - y^2 \longrightarrow$

$$x^2 + y^2 = 8 - x^2 - y^2 \longrightarrow$$

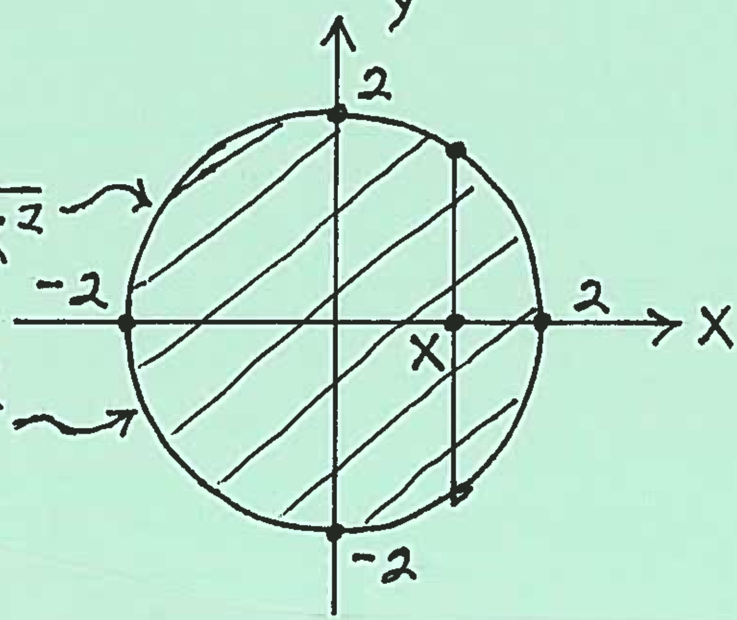
$$2x^2 + 2y^2 = 8 \longrightarrow$$

$$x^2 + y^2 = 4 = 2^2 \longrightarrow$$

$y = \pm\sqrt{4 - x^2}$:

$y = \sqrt{4-x^2}$

$y = -\sqrt{4-x^2}$



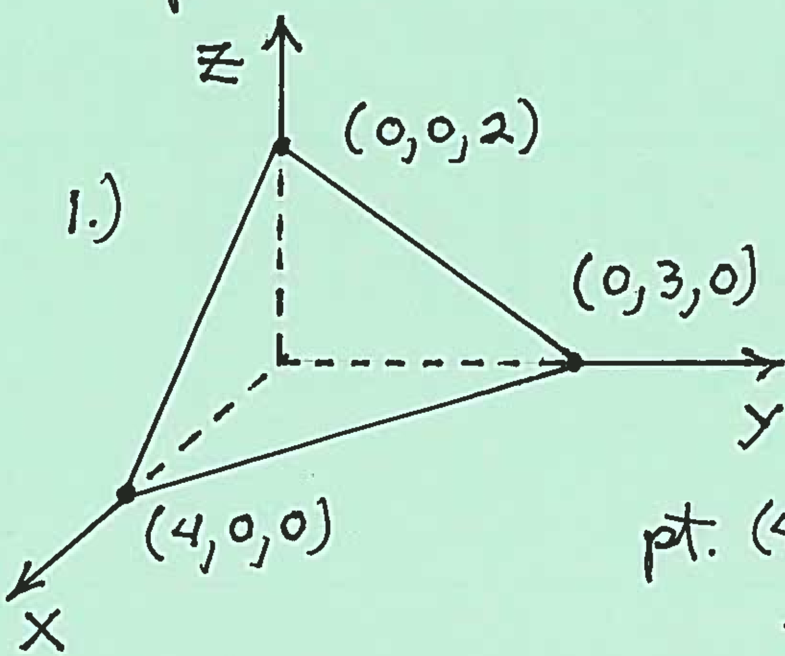
$$R: \begin{cases} -2 \le x \le 2 \\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \\ x^2 + y^2 \le z \le 8 - x^2 - y^2 \end{cases}$$

Example: Consider tetrahedron R with vertices $(0,0,0)$, $(4,0,0)$, $(0,3,0)$ and $(0,0,2)$.

1.) Sketch the solid R.

2.) Find an equation for the surface of R, which is a plane.

1.)



2.) The general equation of a plane is

$ax + by + cz = 1$:

pt. $(4,0,0)$: $a(4)+b(0)+c(0)=1$

$\longrightarrow a = \frac{1}{4}$;

pt. $(0,3,0)$: $a(0)+b(3)+c(0)=1$

$\longrightarrow b = \frac{1}{3}$;

pt. $(0,0,2)$: $a(0)+b(0)+c(2)=1$

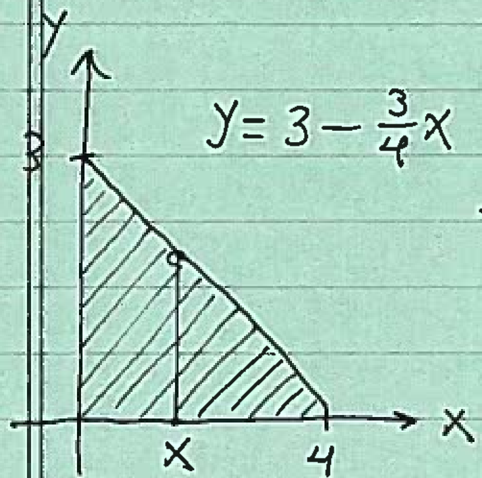
$\longrightarrow c = \frac{1}{2}$, then

$\frac{1}{4}X + \frac{1}{3}Y + \frac{1}{2}Z = 1 \longrightarrow 3X + 4Y + 6Z = 12$

$\longrightarrow Z = \frac{1}{6}(12 - 3X - 4Y)$

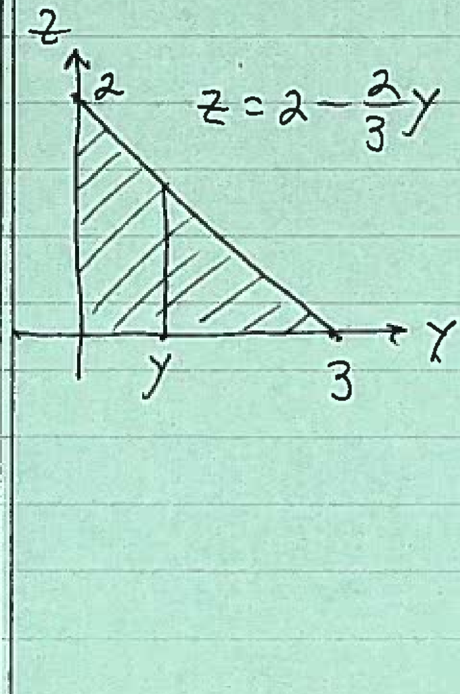
3.) Describe R by 1st projecting R onto the

a.) xy-plane :



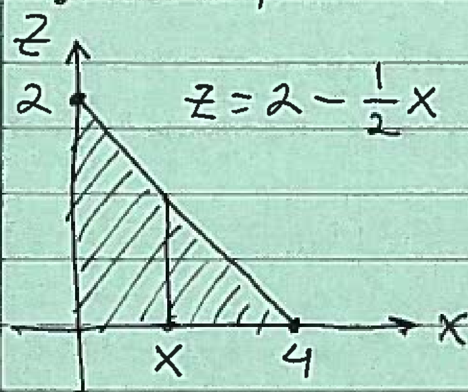
$$\left\{\begin{array}{l} 0 \le x \le 4, \\ 0 \le y \le 3 - \frac{3}{4}x \\ \text{and } 3x + 4y + 6z = 12 \rightarrow \\ \qquad z = \frac{1}{6}(12 - 3x - 4y) \rightarrow \\ 0 \le z \le \frac{1}{6}(12 - 3x - 4y) \end{array}\right.$$

b.) yz-plane :



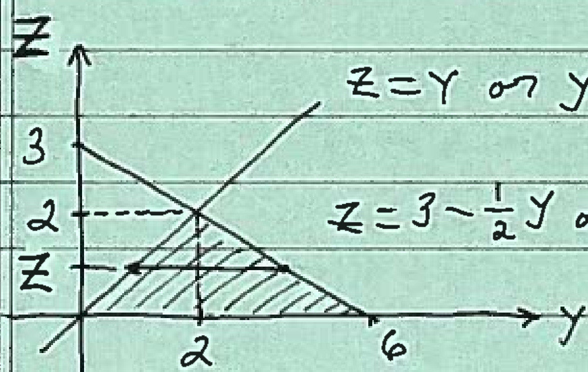
$$\left\{\begin{array}{l} 0 \le y \le 3, \\ 0 \le z \le 2 - \frac{2}{3}y \\ \text{and } 3x + 4y + 6z = 12 \rightarrow \\ \qquad x = \frac{1}{3}(12 - 4y - 6z) \rightarrow \\ 0 \le x \le \frac{1}{3}(12 - 4y - 6z) \end{array}\right.$$

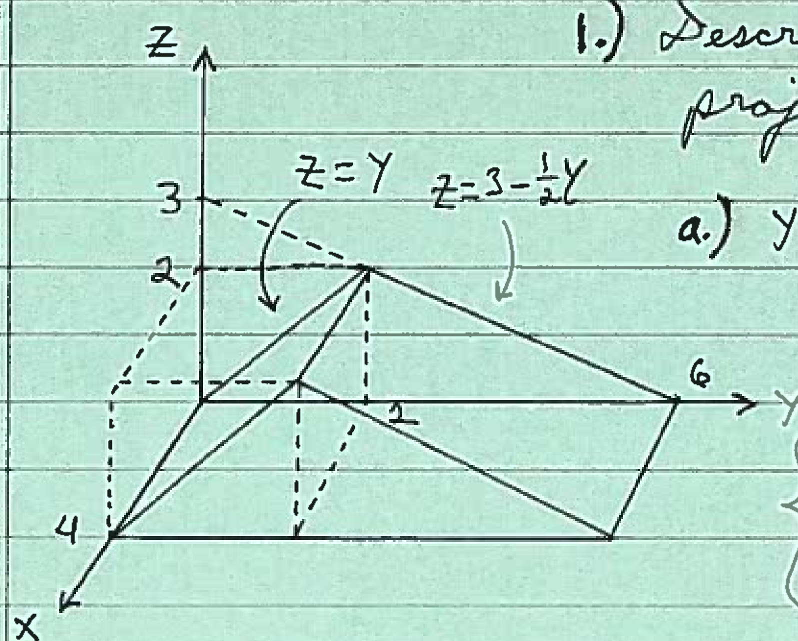
c.) xz-plane:



$$\begin{cases} 0 \le x \le 4, \\ 0 \le z \le 2 - \frac{1}{2}x \\ \text{and } 3x + 4y + 6z = 12 \rightarrow \\ \quad y = \frac{1}{4}(12 - 3x - 6z) \rightarrow \\ 0 \le y \le \frac{1}{4}(12 - 3x - 6z) \end{cases}$$

Example: Consider the solid R bounded by the planes $x=0$, $x=4$, $z=0$, $z=y$, and $z = 3 - \frac{1}{2}y$. Sketch solid.



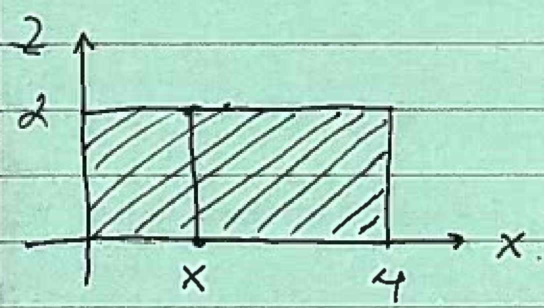


1.) Describe R by 1st projecting R onto the

a.) yz-plane.

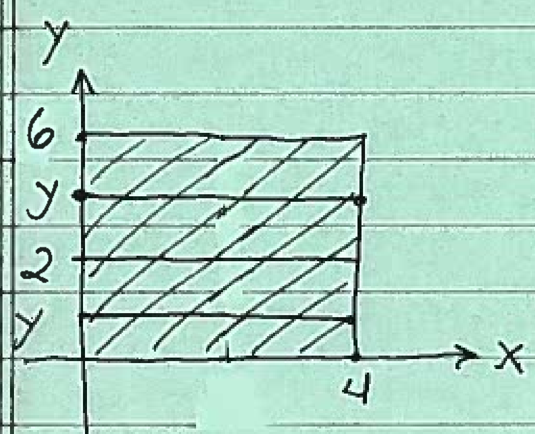
$$\begin{cases} 0 \le z \le 2 \\ z \le y \le 6 - 2z \\ 0 \le x \le 4 \end{cases}$$

b.) xz-plane.



$$\begin{cases} 0 \le x \le 4 \\ 0 \le z \le 2 \\ z \le y \le 6-2z \end{cases}$$

c.) xy-plane.



$$\begin{cases} 0 \le y \le 2 \\ 0 \le x \le 4 \\ 0 \le z \le y \end{cases}$$ and

$$\begin{cases} 2 \le y \le 6 \\ 0 \le x \le 4 \\ 0 \le z \le 3-\frac{1}{2}y \end{cases}$$