

Section 15.1
Thomas Calculus
11th Ed.

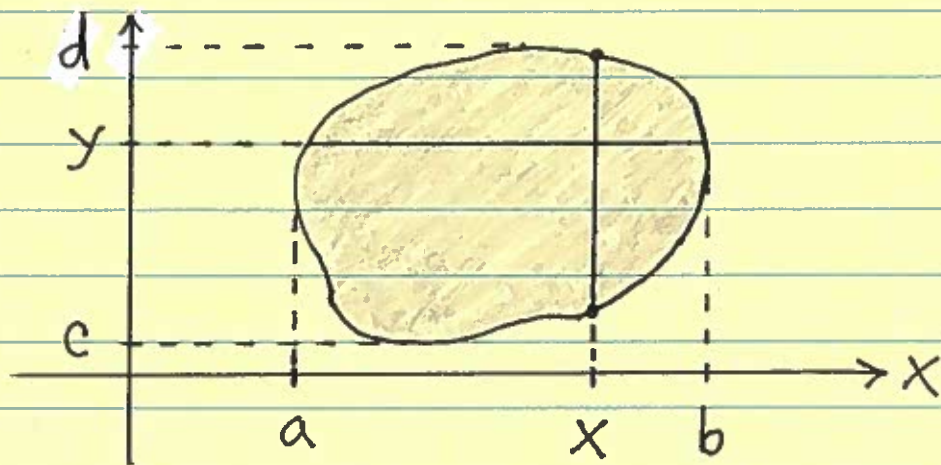
Double Integrals

We will use Double Integrals to compute Area, Volume, Average Value of a Function, Moments, Centroids, Center of Mass, Mass, Moment of Inertia, etc.

Before we learn about Double Integrals, we need to be able to describe flat regions in the xy -plane using

Vertical Cross-Sections
and
Horizontal Cross-Sections.

Describing Regions R in the xy-plane Using Vertical Cross-Sections and Horizontal Cross-Sections

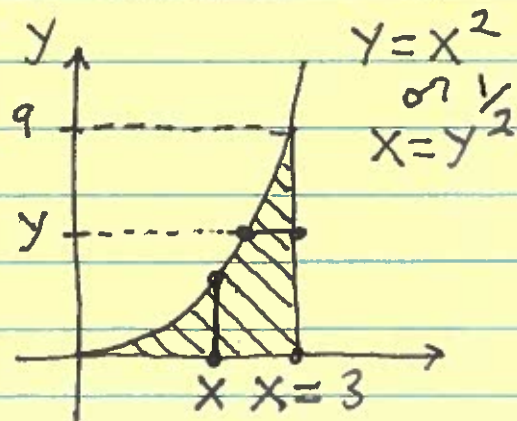


Example: Describe each region R bounded by the graphs of the given equations, using

- vertical cross-sections
- horizontal cross-sections

1.) $R: y=x^2, y=0, x=3$

a.)
$$\begin{cases} 0 \leq x \leq 3 \\ 0 \leq y \leq x^2 \end{cases}$$

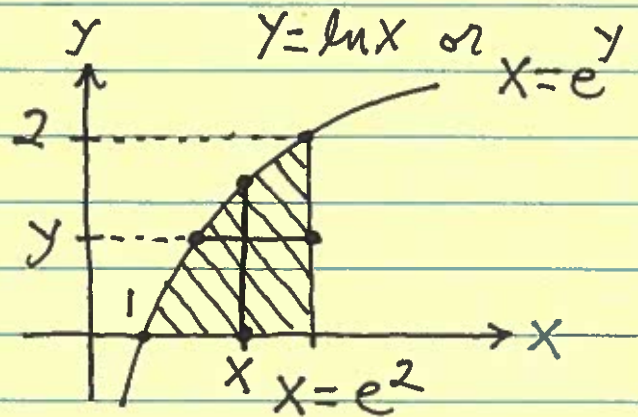


$$b.) \begin{cases} 0 \leq y \leq 9 \\ y^2 \leq x \leq 3 \end{cases}$$

$$2.) R: y = \ln x, y = 0, x = e^2$$

$$a.) \begin{cases} 1 \leq x \leq e^2 \\ 0 \leq y \leq \ln x \end{cases}$$

$$b.) \begin{cases} 0 \leq y \leq 2 \\ e^y \leq x \leq e^2 \end{cases}$$



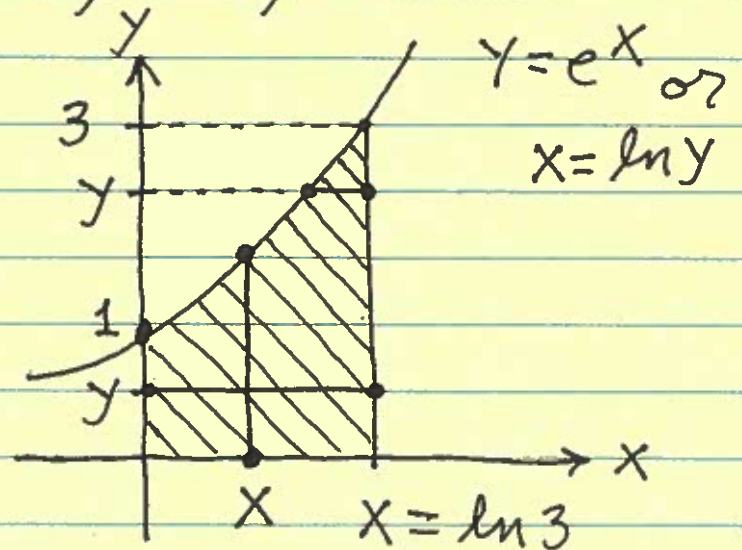
$$3.) R: y = e^x, x = 0, y = 0, x = \ln 3$$

$$a.) \begin{cases} 0 \leq x \leq \ln 3 \\ 0 \leq y \leq e^x \end{cases}$$

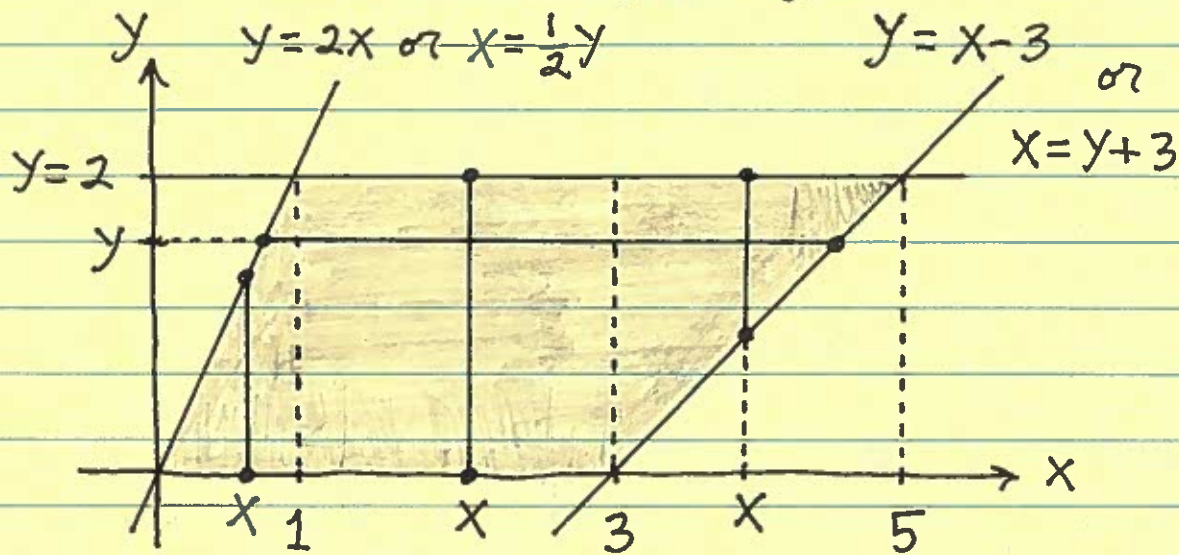
$$b.) \begin{cases} 0 \leq y \leq 1 \\ 0 \leq x \leq \ln 3 \end{cases}$$

and

$$\begin{cases} 1 \leq y \leq 3 \\ \ln y \leq x \leq \ln 3 \end{cases}$$



4.) $R: y=2x, y=x-3, y=2, y=0$



a.) $\begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 2x \end{cases}, \begin{cases} 1 \leq x \leq 3 \\ 0 \leq y \leq 2 \end{cases}, \text{ and } \begin{cases} 3 \leq x \leq 5 \\ x-3 \leq y \leq 2 \end{cases}$

b.) $\begin{cases} 0 \leq y \leq 2 \\ \frac{1}{2}y \leq x \leq y+3 \end{cases}$

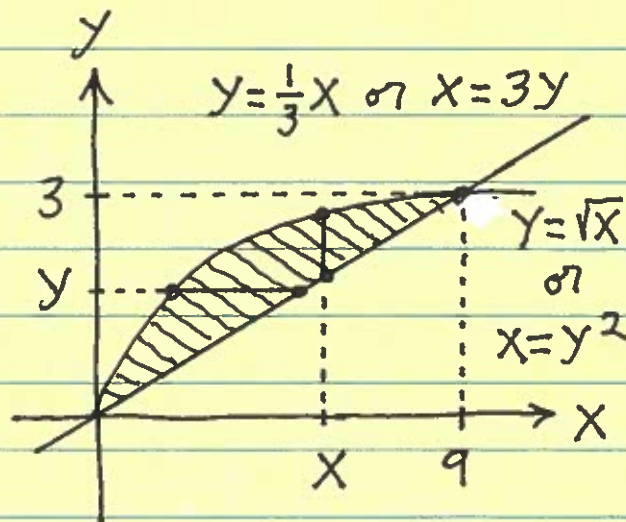
5.) $R: y=\sqrt{x}, y=\frac{1}{3}x$

$$\sqrt{x} = \frac{1}{3}x \rightarrow x = \frac{1}{9}x^2$$

$$\rightarrow 9x = x^2 \rightarrow 0 = x^2 - 9x$$

$$\rightarrow x(x-9) = 0$$

$$\rightarrow x=0, x=9$$



$$a.) \begin{cases} 0 \leq x \leq 9 \\ \frac{1}{3}x \leq y \leq \sqrt{x} \end{cases}$$

$$b.) \begin{cases} 0 \leq y \leq 3 \\ y^2 \leq x \leq 3y \end{cases}$$