1.) Consider the mapping \( F \) given by \( F(u, v) = (3u - 2v, u + v) = (x, y) \). Let \( R \) be the rectangle and its interior in the \( uv \)-plane with vertices \((0, 0), (2, 0), (2, 3), \) and \((0, 3)\).
   a.) Find the image \( S \) of \( R \) under \( F \) and the area of \( S \).
   b.) Find a mapping \( G \) which maps \( S \) to \( R \).

2.) Redo problem 1.) where \( R \) is the triangle and its interior with vertices \((0, 0), (-2, 3), \) and \((2, 0)\).

3.) Consider the mapping \( F \) given by \( F(u, v, w) = (u - v + 2w, 2u + v - w, 3u + 2v + w) = (x, y, z) \). Let \( R \) be the rectangle box and its interior in the \(uvw\)-space with vertices \((0, 0, 0), (2, 0, 0), (2, 3, 0), (0, 3, 0), (0, 0, 4), (2, 0, 4), (2, 3, 4), (0, 3, 4)\).
   a.) Find the image \( S \) of \( R \) under \( F \) and the volume of \( S \).
   b.) Find a mapping \( G \) which maps \( S \) to \( R \).

4.) Plot the curve \( C \) determined by each vector function.
   a.) \( \vec{r}(t) = e^t \vec{i} + e^{2t} \vec{j} \) for \(-1 \leq t \leq 1\)
   b.) \( \vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} \) for \(0 \leq t \leq 2\pi\)
   c.) \( \vec{r}(t) = \sqrt{t} \cos t \vec{i} + \sqrt{t} \sin t \vec{j} \) for \(0 \leq t \leq 4\pi\)
   d.) \( \vec{r}(t) = 2t \vec{i} + 3t \vec{j} + 4t \vec{k} \) for \(0 \leq t \leq 2\)
   e.) \( \vec{r}(t) = \sin t \vec{i} + \cos t \vec{j} + t \vec{k} \) for \(0 \leq t \leq 4\pi\)

5.) Assume that the motion of a particle along path \( C \) is determined by the position function \( \vec{r}(t) = f(t) \vec{i} + g(t) \vec{j} + h(t) \vec{k} \). We know that the speed of motion at time \( t \) is \( \left| \vec{v}(t) \right| = \frac{ds}{dt} = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2} \). Show that the acceleration of motion at time \( t \) is given by \( a(t) = \frac{\vec{v}(t) \cdot \vec{a}(t)}{\left| \vec{v}(t) \right|} \).

6.) Assume that the path \( C \) of a bird in flight is determined by the vector function \( \vec{r}(t) = t \vec{i} + t^2 \vec{j} + 2t \vec{k} \) for \( t \geq 0 \). Find the bird’s position vector, velocity vector, speed, acceleration vector, and acceleration at time
   a.) \( t = 0 \).
   a.) \( t = 1 \).
   a.) \( t = 2 \).

7.) The position of a bicyclist is determined by the vector function \( \vec{r}(t) = (3t) \vec{i} + (3 \sin t) \vec{j} \) for \( 0 \leq t \leq 2\pi \). Determine the bicyclist’s maximum speed.
8.) Find vector function \( \vec{r}(t) \) if \( \vec{r}''(t) = \vec{i} + t \, \vec{j} + \cos 2t \, \vec{k} \), \( \vec{r}'(0) = \vec{i} + \vec{j} + \vec{k} \), and \( \vec{r}(0) = 2\vec{i} - \vec{j} - \vec{k} \).

9.) A super ball is projected at an angle of 75° with initial speed 100 m/sec.
   a.) How high does the ball go?
   b.) How long is the ball in the air?
   c.) How far downrange does the ball travel?

10.) A ball bearing is projected at an angle of 60° and lands 500 feet downrange. What was the ball bearing’s initial speed?

11.) A kiwi is projected at an angle of \( \alpha \) degrees with an initial speed of 100 m/sec. If it lands 200 meters downrange, what is \( \alpha \)?

12.) Assume that \( \vec{u}(t) = a(t) \, \vec{i} + b(t) \, \vec{j} + c(t) \, \vec{k} \), \( \vec{v}(t) = f(t) \, \vec{i} + g(t) \, \vec{j} + h(t) \, \vec{k} \), and \( y = k(t) \).
   a.) (Dot Product Rule) Prove that \( D\{\vec{u}(t) \cdot \vec{v}(t)\} = \vec{u}(t) \cdot \vec{v}'(t) + \vec{u}'(t) \cdot \vec{v}(t) \).
   b.) (Chain Rule) Prove that \( D\{\vec{u}(k(t))\} = \vec{u}'(g(t))k'(t) \).

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

13.) Find the limit of the following sequence of numbers:
   \( 2, 2 - \frac{1}{2}, 2 - \frac{1}{2 - \frac{1}{2}}, 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}, 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}} \cdots \)