1.) Show that each vector field is conservative. Then evaluate the work integral \( \int_C \vec{F} \cdot \vec{T} \, ds \) for each along the given path \( C \).

a.) \( \vec{F}(x, y) = (2xy) \, \vec{i} + (x^2 + y^3) \, \vec{j} \), where \( C \) : curve \( y = x^3(x - 1)^2 \) for \(-1 \leq x \leq 2\)

b.) \( \vec{F}(x, y) = (\sin y) \, \vec{i} + (x \cos y + 3) \, \vec{j} \), where \( C \) : ellipse \( \left( \frac{x}{2} \right)^2 + \left( \frac{y}{3} \right)^2 = 1 \)

c.) \( \vec{F}(x, y) = (2x) \, \vec{i} + (2yz^2) \, \vec{j} + (2y^2z) \, \vec{k} \), where \( C \) : any path from \((0, 0, 0)\) to \((2, 3, 4)\)

2.) Use Green’s Theorems (Theorem 1, 2, or 3 from class) to evaluate each line integral.

a.) \( \int_C \vec{F} \cdot \vec{n} \, ds \), where \( \vec{F}(x, y) = (3x) \, \vec{i} + (2y) \, \vec{j} \) and \( C \) : circle \( x^2 + y^2 = 1 \)

b.) \( \int_C (xy) dy - (x^2y) dx \), where \( C \) : rectangle with vertices \((0, 0)\), \((3, 0)\), \((3, 2)\), and \((0, 2)\)

c.) \( \int_C \vec{F} \cdot \vec{T} \, ds \), where \( \vec{F}(x, y) = (\cos(x + y)) \, \vec{i} + (\sin(x + y)) \, \vec{j} \) and \( C \) : triangle with vertices \((0, 0)\), \((3, 0)\), and \((0, 4)\)

d.) \( \int_C (xy) dx + (e^x) dy \), where \( C \) : line segment joining \((0, 0)\) to \((2, 0)\), then the curve \( y = 2x - x^2 \) from \((2, 0)\) to \((0, 0)\)

e.) \( \int_C \vec{F} \cdot \vec{n} \, ds \), where \( \vec{F}(x, y) = (x - y) \, \vec{i} + (x^2 - 2y) \, \vec{j} \) and \( C \) is given in the diagram below. Assume that the top edge of path \( C \) is \( y = k \), an unknown constant greater than \( 1 \), and that the area of the shaded region is \( 10 \): (HINT: Use Green’s Theorem 3.)
3.) Use the fact that the area of region $R$ enclosed by loop $C$ is given by

$$\text{Area of } R = \frac{1}{2} \int_C (x)dy - (y)dx$$

to find the area inside the ellipse $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$.

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"He who is not courageous enough to take risks will accomplish nothing in life." – Muhammad Ali, former world heavyweight boxing champion