Let \( R \) be a flat region in two-dimensional space and let \( \delta(P) \) be the density of the region at point \( P = (x, y) \).

1.) \[ \text{AREA} : \int_R 1 \, dA \] represents the area of region \( R \).

2.) \[ \text{AVERAGE VALUE} : \frac{1}{\text{Area of } R} \int_R f(x, y) \, dA \] represents the average value of function \( z = f(x, y) \) over region \( R \).

3.) \[ \text{MASS} : \int_R \delta(P) \, dA \] represents the mass of region \( R \).

4.) \[ \text{VOLUME} : \int_R f(P) \, dA \] represents the volume of the solid region defined on region \( R \) with height \( f(P) \) at point \( P \).

5.) \[ \text{MOMENT} : \]
   a.) \[ \int_R (x - a) \delta(P) \, dA \] represents the moment of region \( R \) about the vertical line \( x = a \).
   b.) \[ \int_R (y - b) \delta(P) \, dA \] represents the moment of region \( R \) about the horizontal line \( y = b \).

6.) \[ \text{CENTER OF MASS} , (\bar{x}, \bar{y}) : \]
   a.) \[ \bar{x} = \frac{\int_R x \delta(P) \, dA}{\int_R \delta(P) \, dA} \] represents the \( x \)-coordinate of the center of mass of region \( R \).
   b.) \[ \bar{y} = \frac{\int_R y \delta(P) \, dA}{\int_R \delta(P) \, dA} \] represents the \( y \)-coordinate of the center of mass of region \( R \).

7.) \[ \text{CENTROID} , (\bar{x}, \bar{y}) : \]
   a.) \[ \bar{x} = \frac{\int_R x \, dA}{\int_R 1 \, dA} \] represents the \( x \)-coordinate of the centroid of region \( R \).
   b.) \[ \bar{y} = \frac{\int_R y \, dA}{\int_R 1 \, dA} \] represents the \( y \)-coordinate of the centroid of region \( R \).

NOTE : The formulas for centroid follow immediately from the formulas for center of mass by letting density \( \delta(P) = 1 \).

8.) \[ \text{MOMENT OF INERTIA} : \int_R (\text{distance})^2 \delta(P) \, dA \] represents the moment of inertia of region \( R \), where \( \text{distance} \) refers to the distance from point \( P = (x, y) \) in region \( R \) to either a point or axis (line) of rotation.