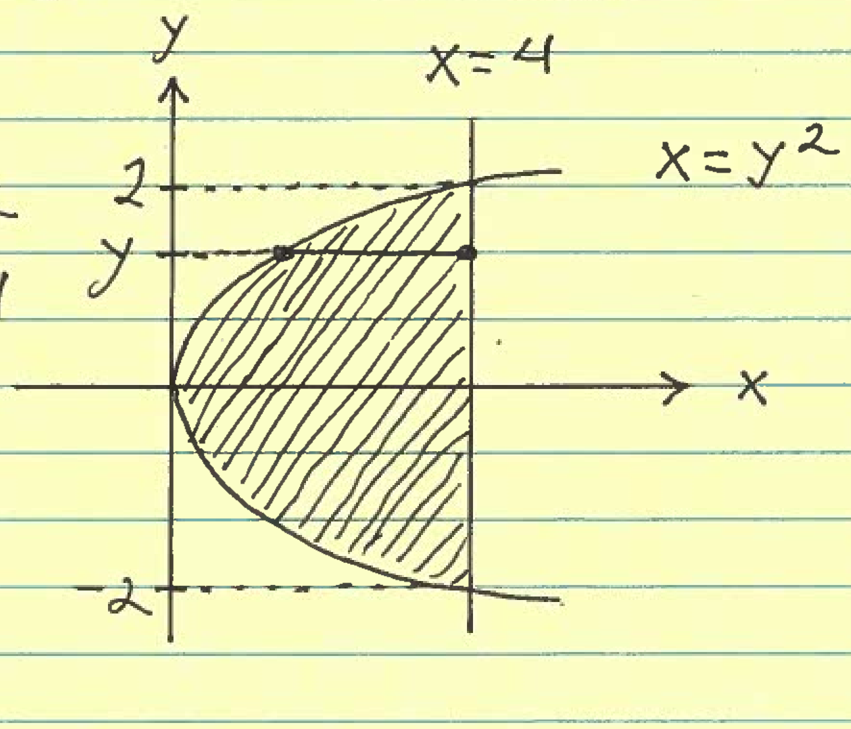
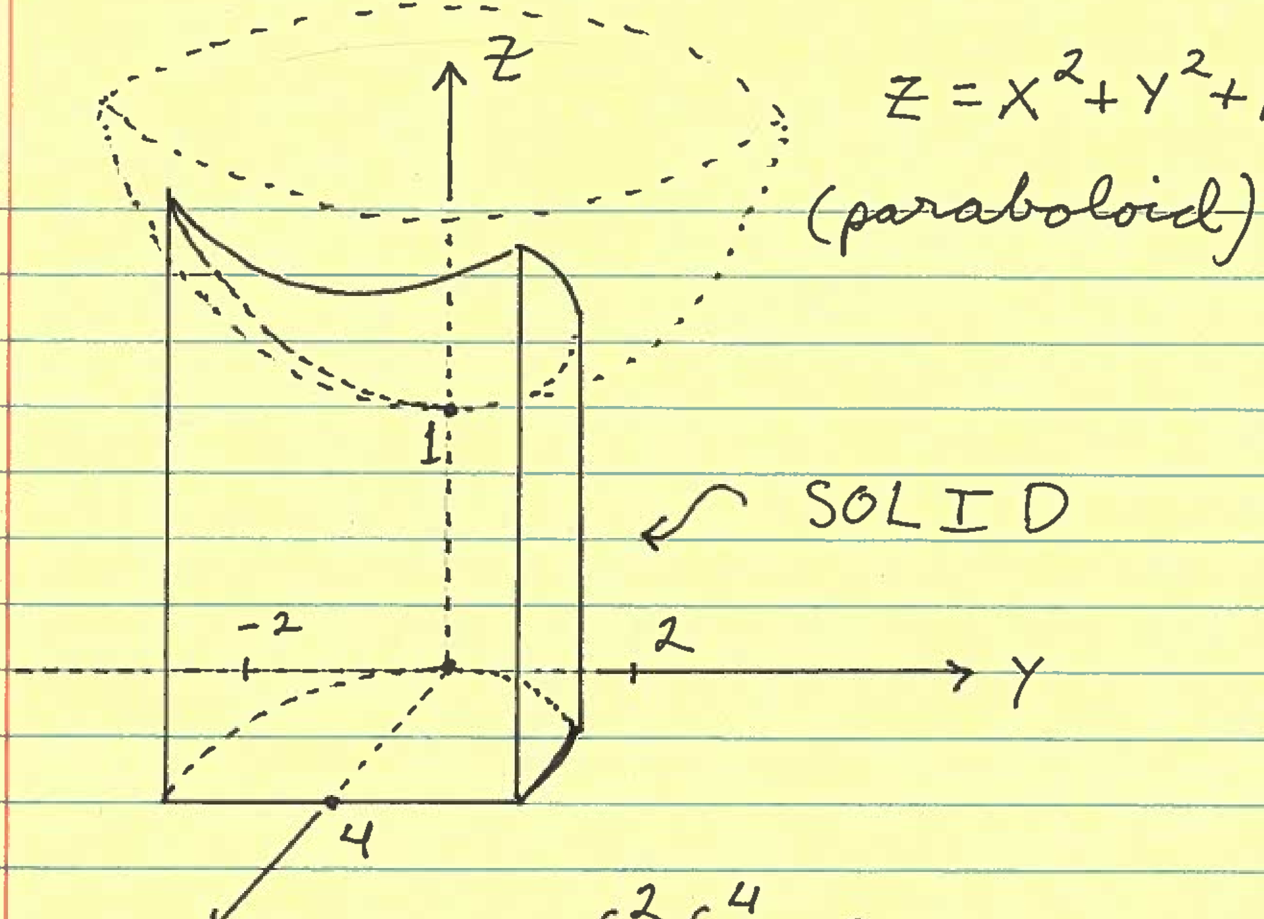
Section 15.1
Thomas Calculus
11th Ed.

Double Integrals, Volume, and Graphing

<u>Example</u>: Find the Volume of the solid beneath the surface $z = x^2 + y^2 + 1$ and above the region R, which is bounded by the graphs of $x = y^2$ and $x = 4$. Sketch the solid. (For Volume SET UP ONLY)

$$R: \begin{cases} -2 \le y \le 2 \\ y^2 \le x \le 4 \end{cases}$$



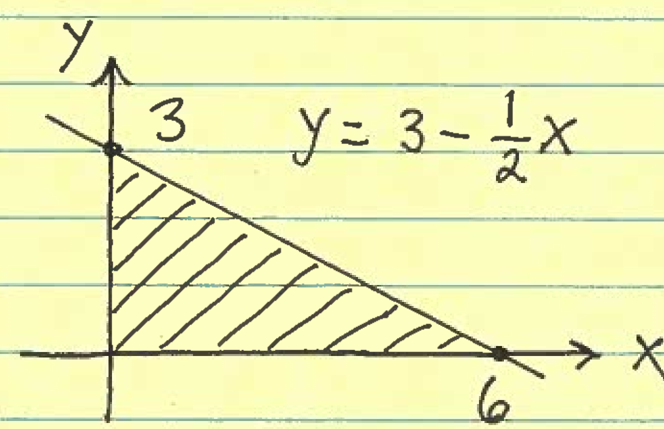


$$\text{VOL} = \int_{-2}^{2}\int_{y^2}^{4} (x^2+y^2+1)\,dx\,dy$$

<u>Example</u>: Sketch the solid whose Volume is given by

$$\text{VOL} = \int_0^6 \int_0^{3-\frac{1}{2}x} \frac{1}{3}(6-x-2y)\,dy\,dx.$$

$$R: \begin{cases} 0 \le x \le 6 \\ 0 \le y \le 3 - \frac{1}{2}x \end{cases}$$



The surface $Z = \frac{1}{3}(6-X-2Y)$

$\longrightarrow \quad 3Z = 6 - X - 2Y$

$\longrightarrow \quad X + 2Y + 3Z = 6$ (plane)

with intercepts

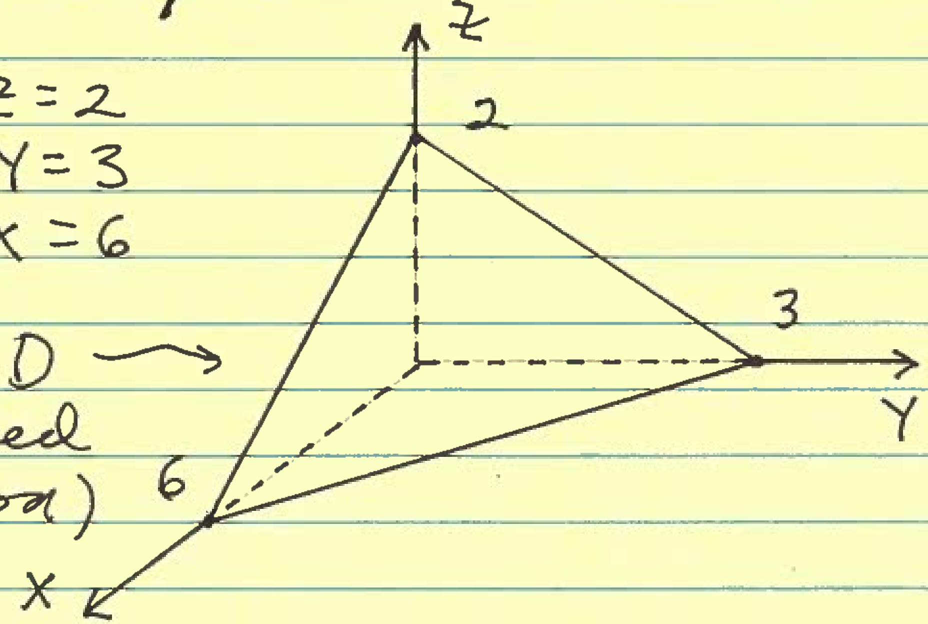
$X=0, Y=0: Z=2$

$X=0, Z=0: Y=3$

$Y=0, Z=0: X=6$

SOLID $\longrightarrow$

(four-sided tetrahedron)



Example: (challenging) Sketch and find the Volume of the solid enclosed by the surfaces

$Z = 4 - X^2 - Y^2$ (paraboloid) and

$Z = 4 - 2Y$ (plane).

Let's first find the projection ("shadow") of this solid onto the XY- plane :

Set the two surface equations equal to each other

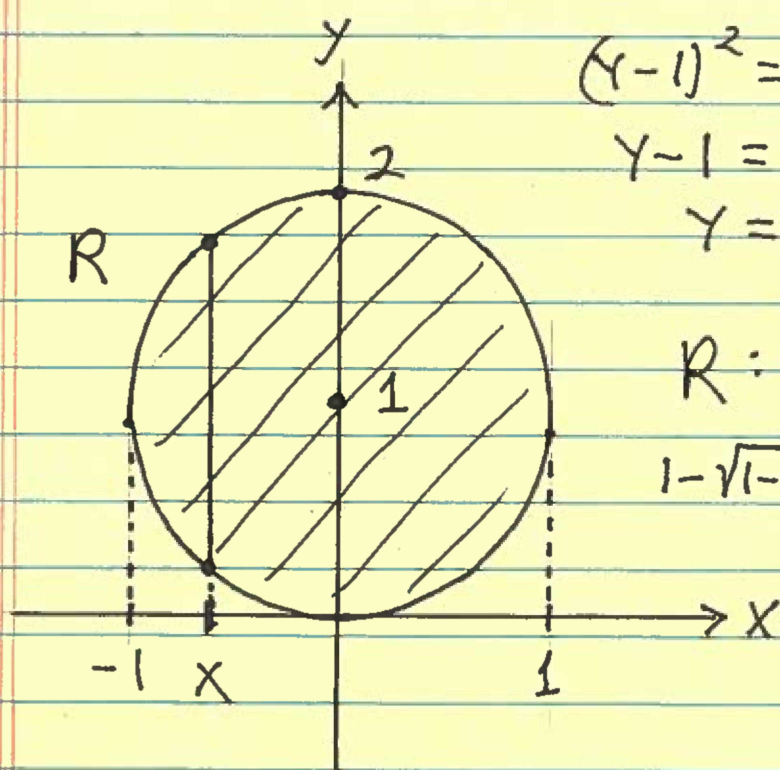
$\rightarrow \quad 4 - X^2 - Y^2 = 4 - 2Y$

$\rightarrow \quad 0 = X^2 + Y^2 - 2Y$ (complete the square)

$\rightarrow \quad 0 = X^2 + (Y^2 - 2Y + 1) - 1$

$\rightarrow \quad X^2 + (Y-1)^2 = 1$

(circle of radius 1, centered at (0,1)), and



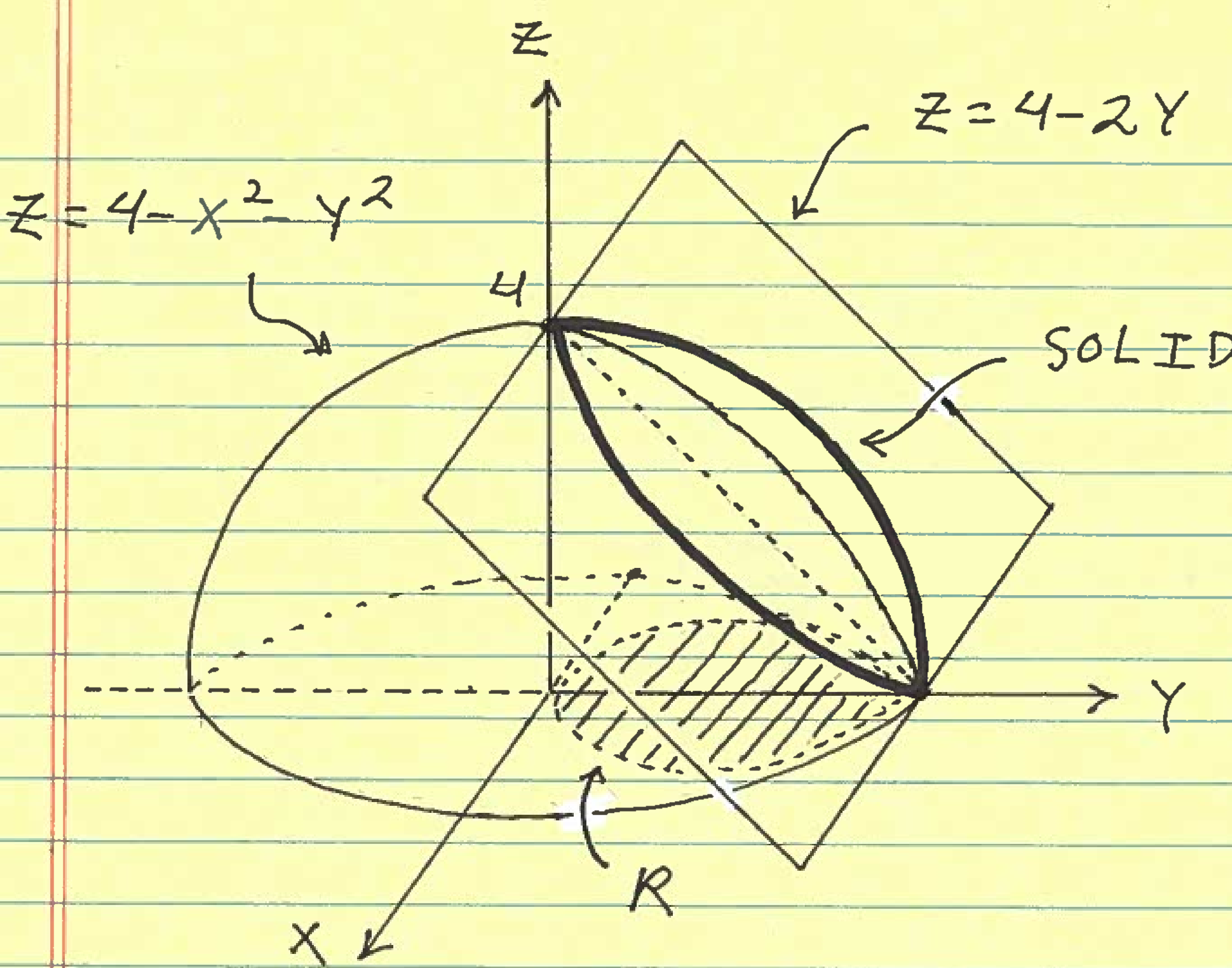
$(Y-1)^2 = 1 - X^2 \rightarrow$

$Y - 1 = \pm\sqrt{1-X^2} \rightarrow$

$Y = 1 \pm \sqrt{1-X^2}$ so

$R: \ -1 \leq X \leq 1$,

$1 - \sqrt{1-X^2} \leq Y \leq 1 + \sqrt{1-X^2}$



$$\text{VOL} = \int_{-1}^{1}\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} (4-x^2-y^2)\,dy\,dx$$

$$- \int_{-1}^{1}\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} (4-2y)\,dy\,dx$$

$$= \int_{-1}^{1}\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \left[(4-x^2-y^2)-(4-2y)\right] dy\,dx$$

$$= \int_{-1}^{1}\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} (2y-x^2-y^2)\,dy\,dx$$