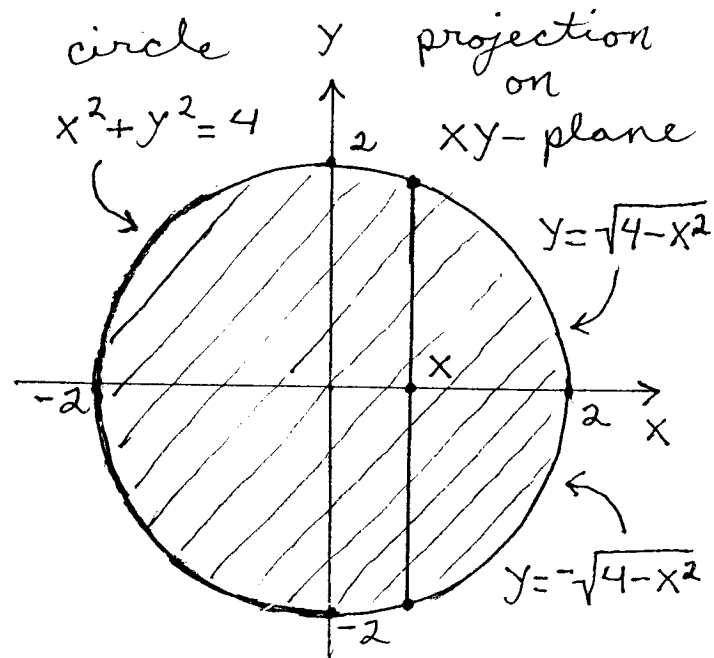
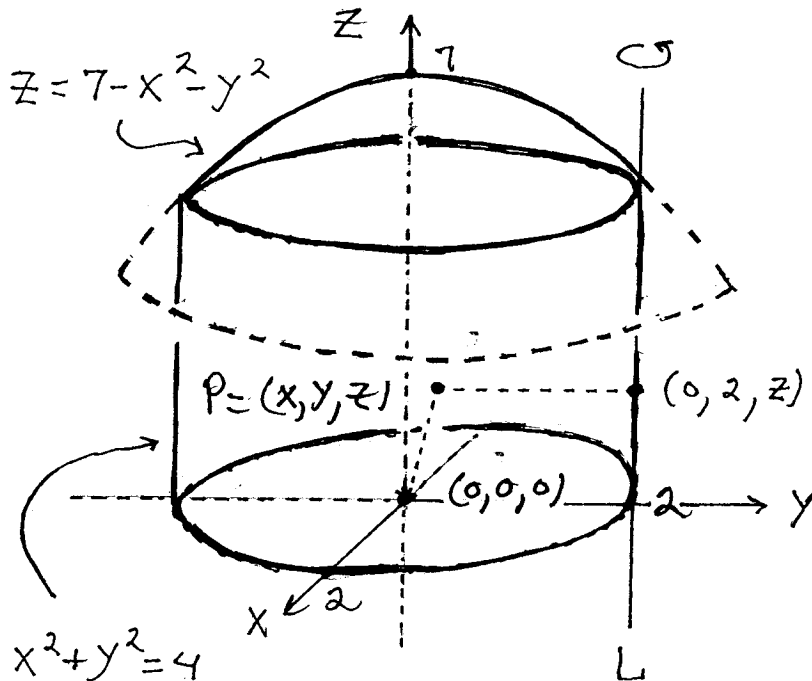


Triple Integral in Rectangular Coordinates, an Example

Consider the solid region R above the plane $z = 0$, inside the cylinder $x^2 + y^2 = 4$, and below the paraboloid $z = 7 - x^2 - y^2$. Assume that the density at point $P = (x, y, z)$ is numerically equal to the distance from P to the origin. SET UP but do not evaluate a triple integral in rectangular coordinates, which represents the *moment of inertia* of R about a line parallel to the z -axis and passing through the edge of the solid.



$$R: \begin{cases} -2 \leq x \leq 2 \\ -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2} \\ 0 \leq z \leq 7-x^2-y^2 \end{cases};$$

density at P is $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$;

distance from P to line L is

$$\begin{aligned} \text{distance} &= \sqrt{(x-0)^2 + (y-2)^2 + (z-z)^2} \\ &= \sqrt{x^2 + (y-2)^2}; \text{ then} \end{aligned}$$

$$M. \text{ of } I. = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{7-x^2-y^2} (\sqrt{x^2 + (y-2)^2})^2 \cdot \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx$$